MATHEMATICS 9

Exponential and Scientific Notation

Distance Learning

Module 2

Alberta
EDUCATION

Mathematics 9

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Mathematics 9
Student Module Booklet
Module 2
Exponential and Scientific Notation
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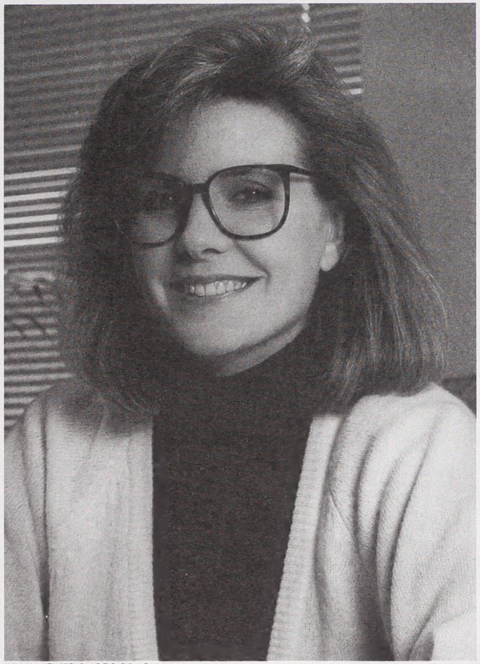
This document is intended for	
Students	✓
Teachers	✓
Administrators	
Parents	
General Public	
Other	

The Learning Technologies Branch has an Internet site that you may find useful. The address is as follows:

http://ednet.edc.gov.ab.ca/ltb

The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

Welcome



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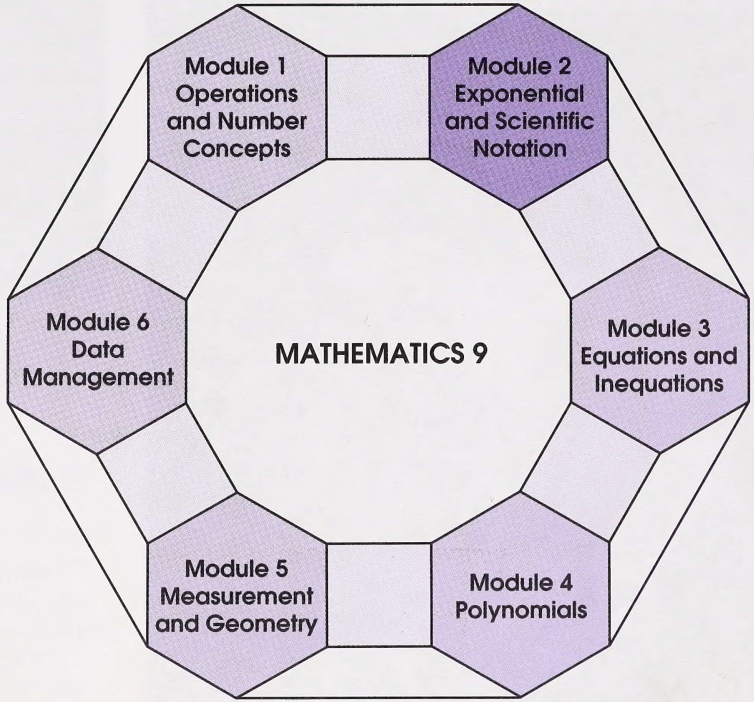
Welcome to Module 2. We hope you'll enjoy your study of Exponential and Scientific Notation.

Mathematics 9 contains six modules. Work through the modules in the order given since several concepts build on each other as you progress in the course.



The document you are presently reading is called a Student Module Booklet. You may find visual cues or icons throughout it. Read the following explanations to discover what each icon prompts you to do.

- Prepare for a problem that will provide a change of topic.

- Prepare for a challenging problem related to the topic of the activity.

- Use the Internet to explore a topic.

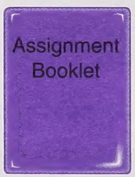
- Use computer software.

- Use a scientific calculator.

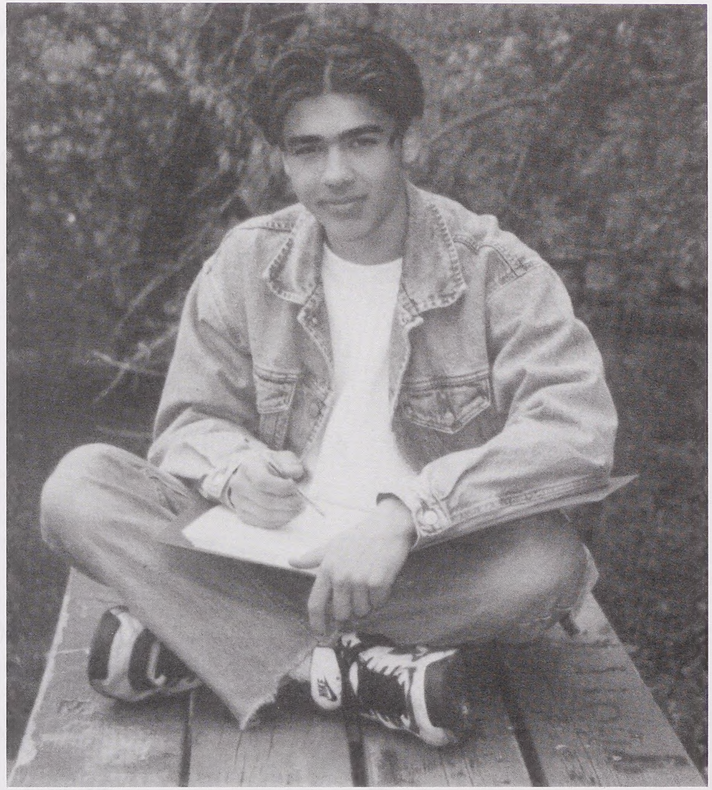
- View a videocassette.

- Pay close attention to important words or ideas.

- Use the suggested answers in the Appendix to correct activities.



- Answer the questions in the Assignment Booklet.



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There are no response spaces provided in this Student Module Booklet. This means that you will need to use your own paper for your responses. You should keep your response pages in a binder so that you can refer to them when you are reviewing or studying.

Technology

Today society is turning to **technology** more than ever before, and it is to your advantage to be able to effectively use technology when required.

Technology is the application of tools, materials, and processes to the solution of problems. More specifically, technology refers to devices and systems that are used in processing, transferring, storing, and communicating information through electronic media.

In Mathematics 9, along with the course materials, you will use a calculator, computer, and videocassette player as tools for learning and doing mathematics.

Calculators are helpful tools for solving problems and exploring patterns and relationships between numbers. Using a calculator will also save you time and help you develop your estimating skills. Therefore, you will be given numerous opportunities in each module to use a calculator.

Computers are useful for organizing and displaying data, or drawing figures. For this reason you will have the chance in many activities to work with popular computer applications such as spreadsheets and draw programs. You will also want to check out the many Internet connections in each module.

Videocassette players allow you to view video programs on key concepts that are difficult to explain in print. That is why video programs are cited in this course.

It is expected that all students will be able to view the video programs and use a calculator, and that most students will do the computer activities. However, if you are unable to access a computer, you may do the calculations using a calculator or draw figures and graphs by hand.

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Problem-Solving Skills

One of the exciting features of this course is that you will develop and improve your ability in problem solving. You will need these problem-solving skills many times in your lifetime. Since this course focuses on problem solving, it is important that you understand what a **problem** is.

A problem is a task for which the method of finding the answer (as well as the answer) is not immediately known.

Like any skill, the skill of problem solving must be developed. Problems may or may not involve computation (adding, subtracting, multiplying, and dividing). Some problems are realistic; others are puzzles.

You will have the opportunity in most activities to try a problem-solving challenge.

The  icon is a cue that the problem will be related to the topic of the activity.

The  icon is a cue that the problem will provide a change of topic.

The Four-stage Process

There are four stages that can be used to solve any problem: understanding the problem, developing a plan, trying the plan, and looking back.

Understanding the Problem

In this stage you should expect to feel puzzled. There are various reasons for feeling this way.

- You may not know the meanings of all the words.
- You may not understand the situation in the problem.
- You may be confused by unnecessary information.

Once you understand the problem, you should think about the problem and make an estimate of what the answer should be. This will help you arrive at a reasonable answer.

Developing a Plan

This is where you should decide on the plan of action that you are going to take to solve the problem. You may consider the following strategies:

- changing your point of view
- using objects
- using diagrams
- making an organized list
- using Venn diagrams
- making a table
- guessing, checking, and revising
- acting out a problem
- working backwards
- simplifying a problem
- finding and applying a pattern
- using elimination
- using truth tables
- using an equation

Note: The Appendix in this module explains these strategies in detail. When you see a problem-solving icon in any module, you should turn to this Appendix and review the problem-solving strategies.

Trying the Plan

In this stage you should try the plan and see if it works.

Be sure to work carefully and record your progress. You are encouraged to use a calculator to help with your calculations.

Note: While trying the plan, you should monitor your progress in order to determine if your plan will lead to a solution. You may find that the plan will not produce a solution, in which case a new plan will have to be developed.

Looking Back

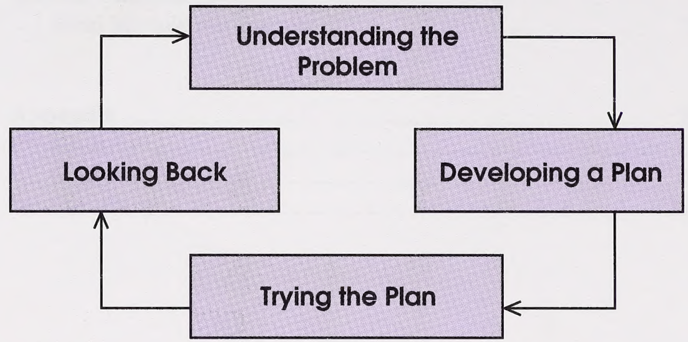
In this stage you should look back at the problem and compare your answer to the estimate you made in the first stage. Restate the problem using your answer.

Ask yourself these questions: "Did my plan work? Is my answer reasonable?"

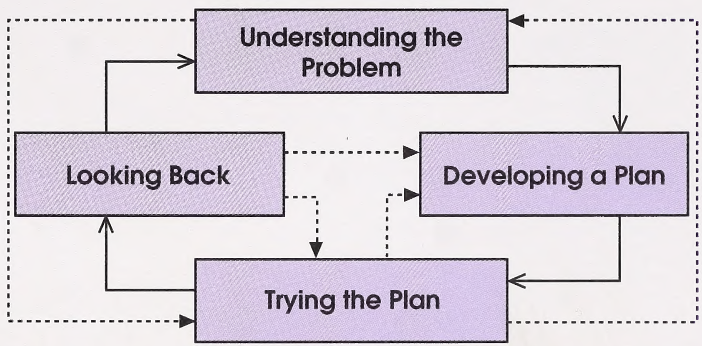
If you did not arrive at an answer, another strategy may work better. If your answer is unreasonable, you may have made errors while trying your plan.

Sequence of Stages

You usually approach a problem in the order outlined in the following diagram.



If you encounter difficulties in your original plan, or if you realize that another strategy will have better results, you may need to return to an earlier stage or use the stages in a different sequence.



CONTENTS

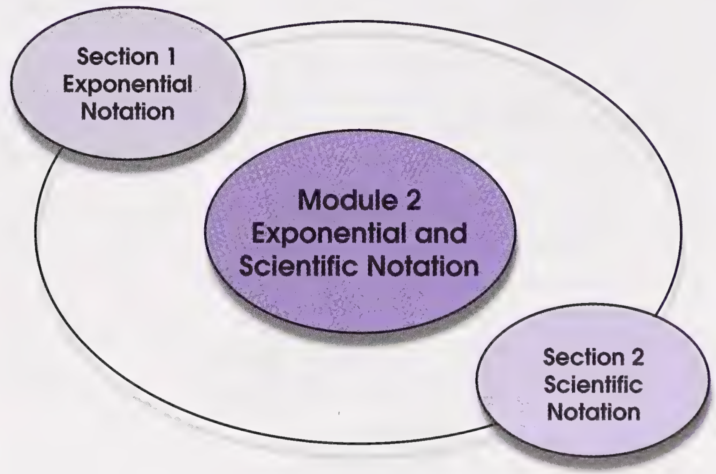
Module Overview

In some ways you can say that the world is getting smaller. You can travel long distances in much shorter times. It no longer takes days, weeks, or months to get around the world—you can now get around the world in a few hours. Voice messages, electronic messages, and other electronic signals are sent to the other side of the world in seconds. The physical sizes of some things are also getting smaller. There are 5.5 million transistors on a computer chip half the size of your finger nail. Can you imagine the size of a single transistor?

On the other hand, the world is expanding beyond Earth. With manned flights to the moon and unmanned flights to other planets, horizons are being expanded to new dimensions. Powerful telescopes enable you to reach out large distances to explore new worlds.

Whether you are considering the microscopic world of research laboratories or the large expanse of the growing universe, you come in contact with very small and very large numbers.

In this module, you will explore ways of expressing these numbers in compact, more manageable forms. You will also identify patterns that will allow you to generalize rules for working with these numbers.



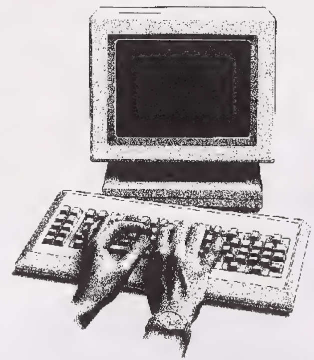
Evaluation

Your mark for this module will be determined by how well you complete the assignments at the end of each section and at the end of the module. In this module you must complete three assignments. The mark distribution is as follows:

Section 1 Assignment	**35 marks**
Section 2 Assignment	**25 marks**
Final Module Assignment	**40 marks**
TOTAL	**100 marks**

When doing the assignments, work slowly and carefully. You must do each assignment independently, but if you are having difficulties, you may review the appropriate section in this module booklet.

If you are working on a CML terminal, you will have a module test as well as a module assignment.



Note

There is a final supervised test at the end of this course. Your mark for the course will be determined by how well you do on the module assignments and the supervised final test.

Section 1: Exponential Notation

Jennifer is doing a project on one of the computers at her father's business. The computer she is using has an SVGA monitor capable of producing over 16 million colours by mixing the three colours—red, green, and blue. Since the monitor is capable of producing 256 tones of each primary colour, there are actually 16 777 216 colours available to Jennifer. How does someone determine the number of colours produced by an SVGA colour monitor?

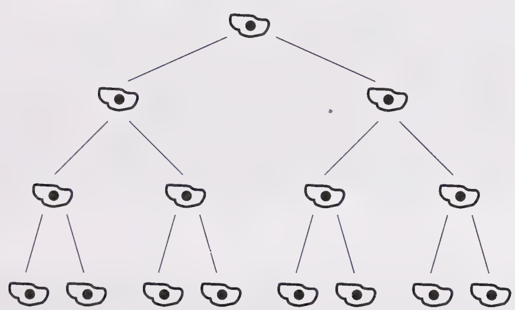
The most common way is to simply multiply 256 and 256 and 256. But, there is another method. It involves using exponential notation—256 to the power of 3 (or 256^3). Exponential notation provides an individual with a more efficient way of writing an equation and a simpler way of writing an expression. (For instance, SVGA monitors can produce 256^3 colours.)

Throughout this section you will work with exponential notation. You will review and increase your knowledge of powers, develop rules for calculating powers by recognizing patterns, and calculate solutions to questions and problems involving powers.

Activity 1: The Basics of Powers

NASA

An amoeba is a single-celled animal that reproduces by a process called fission. One cell divides into two cells; then each of these cells divide into two more cells. In a few hours, a single amoeba can become a colony of amoebas.



To visualize the increase in amoebas, make a model by performing the following steps.

Step 1: Obtain a large rectangular sheet of paper. (You may use a newspaper page.)

Step 2: Fold the sheet in half. The fold represents a division of an amoeba. Each sheet resulting from the fold represents an amoeba.

Step 3: Continue to fold the paper in half until several layers are formed.

1. Copy and complete the following chart by counting the number of layers of paper after each fold.

Number of Folds	Number of Layers
0	1
1	2
2	4
3	
4	

2. Amoeba fission follows the same pattern as the folding paper example. Use the folding paper example to predict the number of amoebas there are after the following:

a. no divisions **b.** one division **c.** two divisions

d. three divisions **e.** four divisions **f.** five divisions

If you have access to a computer and a spreadsheet program (like *ClarisWorks*™); then answer questions 3 and 4 using the program. Otherwise, complete questions 3 and 4 using paper and pencil.

3. Create a chart (similar to the one in question 1) showing the number of amoebas after each division.

If you are using *ClarisWorks*™, follow these steps. **Note:** You may need to consult the user's guide for some assistance.

Step 1: Under the File menu, select New.

Step 2: In the New Document window, select Spreadsheet; then click OK.

Step 3: In cell A1, insert the heading Number of Divisions. You will need to adjust the width of Column A in order to fit the heading.

To adjust the column width, go under the Format menu and select Column Width.

Step 4: In cell B1, insert the heading Number of Amoebas. You will need to adjust the width of column B in order to fit the heading.

Step 5: Enter the numbers 0 to 5 (cells A2 to A7) under the heading in column A.

Step 6: Enter the appropriate number of amoebas under the heading in column B.

Step 7: Go under the File menu and select Save As to save your work.

4. Graph the values given in the chart in question 3. Place Number of Divisions on the horizontal axis and Number of Amoebas on the vertical axis.

If you are using *ClarisWorks*™, follow these steps.

Step 1: Select the data you wish to graph by clicking on cell A1, pressing and holding the shift key, and letting go on cell B7.

Step 2: Under the Options menu, select Make Chart.

Step 3: In the Chart Options window, choose X-Y Line.

Step 4: Click on Axes. Label each axis and choose an appropriate scale.

Step 5: Click on Series. Choose the type of symbol you wish to use to represent each point.

Step 6: Click on Labels. Title the graph Amoeba Fission. **Note:** If there is an X in the box beside Legend, then click in the box to remove it.

Step 7: Click on General. Set the chart range to cover all of the data (A1. . .B7). This may be already set by the program.

Step 8: Click OK. The graph should be displayed.

Step 9: Save the graph in the same file as the chart in question 3.

Note: You may wish to print the chart and the graph together.

Check your answers by turning to the Appendix.

Previously, you used a number in standard form to represent the number of amoebas after each split. **Powers** can also be used to describe the number of amoebas after each division.

A power is an expression written with a **base** and an **exponent**. The base of a power shows the factor that is repeatedly multiplied. The exponent of a power shows the number of times the base is used as a factor. For example, 3^5 is a power where the base is 3 and the exponent is 5.

5. The following chart shows how to express the number of amoebas in different ways.

Number of Divisions	Number of Amoebas at Each Division		
	Standard Form	Factored Form	Power Form
1	2	2	2^1
2	4	2×2	2^2
3	8	$2\times2\times2$	
4	16	$2\times2\times2\times2$	
5	32	$2\times2\times2\times2\times2$	

a. Complete the chart.

b. How many amoebas will there be after six divisions?

c. Written as a power of 2, how many amoebas will there be after six divisions?

d. What is the factored form of 2^6?

e. What is the relationship between the number of factors in the factored form and the exponent in the power form?

Check your answers by turning to the Appendix.

A number in standard form can be written as a power with a given base. Simply find how many times the base can be written as a factor of the number.

Example

Write 125 as a power with a base of 5.

Solution

$$125 = 5\times5\times5$$
$$= 5^3$$

There are three factors of 5; so, the exponent is 3.

6. Write each of the following as a power of 2.

a. 16 **b.** 8 **c.** 32

7. Write each of the following as a power of 3.

a. 9 **b.** 81 **c.** 27

8. Express each of the following as a power.

a. 49 **b.** 144 **c.** 27 **d.** 10 000

Check your answers by turning to the Appendix.

Powers can be read in different ways. For instance, in the statement "The number of amoebas after five divisions is 2^5," 2^5 can be read as follows:

- two to the fifth power
- two exponent five
- the fifth power of two

Note: These are just a few ways of expressing 2^5 in words.

9. How would you read each of these?

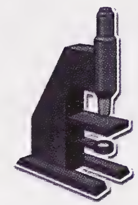
a. 7^3 **b.** $(-6)^4$ **c.** 1.5^5

Check your answers by turning to the Appendix.

Zero Exponents

10. **a.** How many amoebas are there when no divisions have occurred (zero divisions)?

b. What power can be used to describe the number of amoebas after zero divisions? Explain how you arrived at your answer.



Check your answers by turning to the Appendix.

The answers to questions 10.a. and 10.b. show that $2^0 = 1$. You can use patterns to show that other bases with a zero exponent also equal one.

Example 1

What is 5^0?

Solution

You can show that $5^0 = 1$ by using patterns.

Power Form	Standard Form	Pattern
5^4	625	Divide by 5.
5^3	125	Divide by 5.
5^2	25	Divide by 5.
5^1	5	Divide by 5.
5^0	1	

Dividing the standard form by 5 for each reduction in power shows that $5^0 = 1$.

You can also use a scientific calculator to find the value of 5^0. Use $\boxed{x^y}$ (the power key) and enter the following keystrokes. **Note:** On some calculators, the power key may be $\boxed{y^x}$.

$\boxed{5}$ $\boxed{x^y}$ $\boxed{0}$ $\boxed{=}$

$\boxed{1.}$

Therefore, $5^0 = 1$.

Example 2

What is the value of 3^0?

Solution

Power Form	Standard Form	Pattern
3^4	81	Divide by 3.
3^3	27	Divide by 3.
3^2	9	Divide by 3.
3^1	3	Divide by 3.
3^0	1	

From the pattern, $3^0 = 1$.

Use a scientific calculator to confirm the answer. Enter the following keystrokes.

3 x^y 0 =

1.

Therefore, $3^0 = 1$.

11. Enter 0^0 into a scientific calculator using the power key. What is the result?

Check your answers by turning to the Appendix.

Mathematicians say 0^0 is undefined. Therefore, a rule for any non-zero base to the exponent zero can be stated as follows:

Any non-zero base raised to the exponent zero is equal to one.

12. Use the rule to find the value of each of the following.

a. 2^0 **b.** $(-12)^0$ **c.** $\left(\frac{1}{2}\right)^0$ **d.** $3^0 \times 4$

Check your answers by turning to the Appendix.

Negative Exponents

You used the example of the division of an amoeba to establish a pattern for powers. You can extend the patterns for powers and discover the meaning of a power with a negative exponent.

13. What does 2^{-4} mean? What is its value? To find out, copy and complete the following chart into your notebook. The first few have been completed for you.

Power Form	Standard Form	Pattern
2^2	4	Divide by 2.
2^1	2	Divide by 2.
2^0	1	Divide by 2.
2^{-1}	$\frac{1}{2}$	Divide by 2.
2^{-2}	$\frac{1}{4}$	Divide by 2.
2^{-3}	$\frac{1}{8}$	Divide by 2.
2^{-4}		Divide by 2.
2^{-5}		Divide by 2.
2^{-6}		

14. What is the relationship between the standard forms for each of the following pairs of powers?

a. 2^{2} and 2^{-2} **b.** 2^{1} and 2^{-1}

15. What is the relationship between the exponents of each of the pairs of powers in question 14?

After completing questions 13 to 15, you should be able to see that $2^{-4} = \frac{1}{2^{4}} = \frac{1}{16}$.

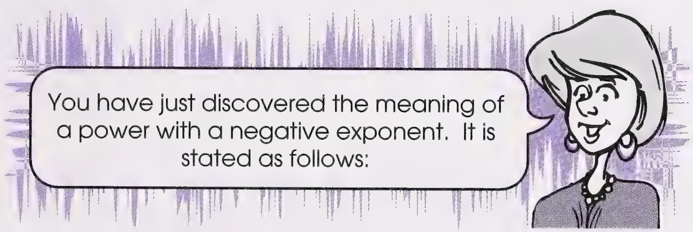
16. Express each of the following with a positive exponent.

a. 4^{-3} **b.** 7^{-4} **c.** $(0.6)^{-5}$

17. Express each of the following with a negative exponent.

a. $\frac{1}{8}\times\frac{1}{8}\times\frac{1}{8}$ **b.** $\frac{1}{(-3)\times(-3)\times(-3)\times(-3)}$

Check your answers by turning to the Appendix.

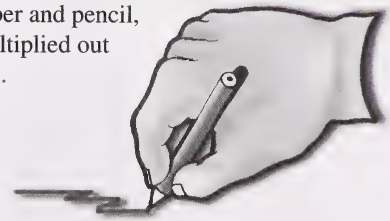


Any non-zero base raised to a negative exponent equals the reciprocal of the base raised to the positive (opposite) exponent.

Evaluating Powers

To evaluate a power is to write the power in standard form. You can evaluate a power using paper and pencil or using a scientific calculator.

To evaluate a power using paper and pencil, write the power as its base multiplied out as many times as it has factors.



Example 1

Evaluate 5^{4}.

Solution

$$\begin{aligned} 5^{4} &= 5\times5\times5\times5 \\ &= 25\times5\times5 \\ &= 125\times5 \\ &= 625 \end{aligned} \quad \text{or} \quad \begin{aligned} 5^{4} &= 5\times5\times5\times5 \\ &= (5\times5)\times(5\times5) \\ &= 25\times25 \\ &= 625 \end{aligned}$$

It is sometimes helpful to group factors rather than work from left to right.

18. Evaluate each of the following using paper and pencil.

a. 5^6 **b.** 13^3 **c.** 39^4 **d.** 1.7^3

Check your answers by turning to the Appendix.

To evaluate a power using a scientific calculator, you can either use the automatic constant (× =) or the power key (x^y or y^x).

Example 2

Evaluate 8^4.

Solution

Method 1: Using the Automatic Constant

8 × ×

8.

=

64.

=

512.

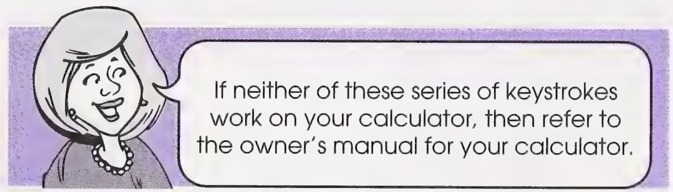
=

4096.

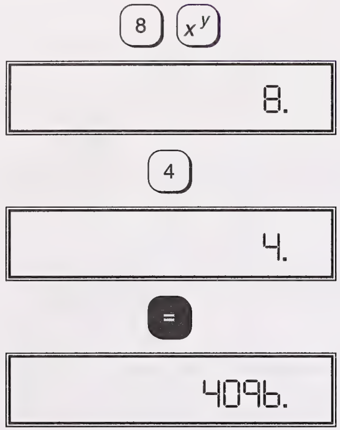
Therefore, $8^4 = 4096$.

Note: Many scientific calculators do repetitive calculations by storing a number and its associated operation for repeated use. The data is stored by pressing the constant, 8 , and then the operation, × , twice. Once the data is stored, you then only use = . In this example, = is pressed three times in order to calculate 8^4.

The automatic constant feature is not the same for all calculators. For example, some scientific calculators require you to press × only once, and then = .



Method 2: Using the Power Key



Therefore, $8^4 = 4096$.

Use a scientific calculator to answer question 17.

19. Evaluate the following powers. First, use the automatic constant; then use the power key. **Remember:** Read the exponential notation correctly.

a. 6^5 **b.** 7^4 **c.** 1.2^3

20. Which method do you prefer? Why?

Check your answers by turning to the Appendix.

Now try evaluating a fraction to a given exponent. Using paper and pencil, multiply out the factors in the numerator and the factors in the denominator.

Example 3

Evaluate $\left(\frac{3}{5}\right)^4$.

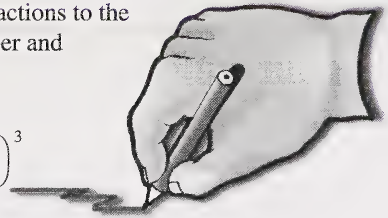
Solution

$$\begin{aligned}\left(\frac{3}{5}\right)^4 &= \frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}\\ &= \frac{9}{25}\times\frac{3}{5}\times\frac{3}{5}\\ &= \frac{27}{125}\times\frac{3}{5}\\ &= \frac{81}{625}\end{aligned}$$

or

$$\begin{aligned}\left(\frac{3}{5}\right)^4 &= \frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}\\ &= \frac{9}{25}\times\frac{9}{25}\\ &= \frac{81}{625}\end{aligned}$$

21. Evaluate the following fractions to the given exponent using paper and pencil.



a. $\left(\frac{2}{3}\right)^5$ **b.** $\left(\frac{4}{7}\right)^3$

c. $\left(3\frac{1}{3}\right)^2$

Check your answers by turning to the Appendix.

To evaluate a fraction to an exponent using the automatic constant, you need to have $a\frac{b}{c}$ (a fraction key) on your calculator. If you have such a key, enter the following keystrokes using the power in Example 3.

3 $a\frac{b}{c}$ 5 × ×

3 ⌟ 5. ← This means $\frac{3}{5}$.

=

9 ⌟ 25. ← This means $\frac{9}{25}$.

=

27 ⌟ 125. ← This means $\frac{27}{125}$.

=

81 ⌟ 625. ← This means $\frac{81}{625}$.

$\therefore \left(\frac{3}{5}\right)^4 = \frac{81}{625}$

Note: Not all calculators have the fraction key. In those that do, the calculator display capacity may be limited to three digits in the numerator or denominator. When an answer exceeds this, the fraction automatically is converted to the decimal equivalent.

If you do not have a fraction key, then convert the fraction to a decimal first; then proceed using the automatic constant.

A fraction to a given exponent can also be evaluated using x^y. The following steps show the power in Example 3 evaluated using the power key.

3 $a\frac{b}{c}$ 5 x^y 4 =

0. 1296

$\frac{81}{625} = 0.1296$

Therefore, $\left(\frac{3}{5}\right)^4 = 0.1296$ or $\frac{81}{625}$.

Notice that the answer is displayed as a decimal when the power key is used.

22. Evaluate the following fractions using a calculator. Use either the automatic constant or the power key. Round your answers to three decimal places.

a. $\left(\frac{1}{3}\right)^4$ **b.** $\left(\frac{3}{4}\right)^5$ **c.** $\left(2\frac{3}{8}\right)^3$

23. Use your answers to question 20 to answer the following:

a. Which method did you prefer?

b. What step do you have to do mentally when using the automatic constant?

c. Which method is more efficient in terms of the number of keystrokes entered?

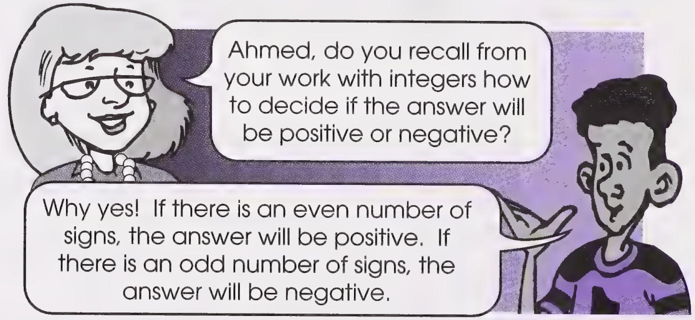
24. Indicate which is greater in each of the following pairs of powers.

a. 3^2 or 2^3 **b.** $\left(\frac{1}{2}\right)^3$ or $\left(\frac{1}{3}\right)^2$

Check your answers by turning to the Appendix.

Thus far, you have evaluated positive rational bases. Next, you will evaluate negative rational bases. Be sure to pay attention to whether there is an odd or even number of negative signs.

When using paper and pencil to evaluate a negative base to a given exponent, write out the factors. You can then count the number of signs and decide if the answer will be positive or negative.



Example 4

Evaluate $(-5)^4$.

Solution

$$\begin{aligned}(-5)^4 &= (-5)\times(-5)\times(-5)\times(-5)\\ &= +(5\times5\times5\times5)\\ &= +625\end{aligned}$$

There are four negative signs; thus, the answer is positive.

Example 5

Evaluate $(-0.4)^3$.

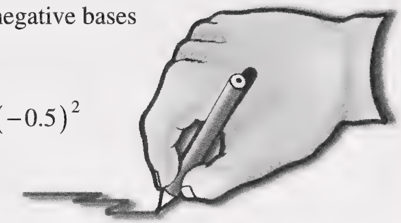
Solution

$$\begin{aligned}(-0.4)^3 &= (-0.4)\times(-0.4)\times(-0.4)\\ &= -(0.4\times0.4\times0.4)\\ &= -0.064\end{aligned}$$

There is an odd number of negative signs; thus, the answer is negative.

25. What way, other than counting the number of factors, can you think of to determine the sign of an answer?

26. Evaluate the following negative bases using paper and pencil.



a. $(-6)^3$ **b.** $(-0.5)^2$

c. $(-3)^5$

Check your answers by turning to the Appendix.

You can also use a scientific calculator to evaluate powers with negative bases. You can use either the automatic constant or the power key.

When using the automatic constant to evaluate Example 4, your keystrokes and display will look as follows:

5 +/− × ×

-5.

=

25.

=

- 125.

=

625.

$\therefore\ (-5)^4 = 625$

27. What does +/− do?

Check your answers by turning to the Appendix.

To use the power key on a scientific calculator to evaluate Example 4, enter the following keystrokes.

5 +/− x^y 4 =

625.

$\therefore\ (-5)^4 = 625$

It is easy to make an error when pressing keys on a calculator; so, always estimate your answer first to be sure the display is reasonable. Be particularly careful with the sign. **Remember:** If the exponent is even, there will be an even number of factors and the answer will be positive; if the exponent is odd, there will be an odd number of factors and the answer will be negative.

28. Use the power key to evaluate the following. Round your answer to three decimal places where necessary.

a. $(-4)^5$ **b.** $(-0.85)^3$ **c.** $(-1.7)^4$

Check your answers by turning to the Appendix.

Opposites

Off is the opposite of on. Turning a screw to the left is opposite of turning it to the right. Negative is the opposite of positive.

In previous mathematics courses you discovered that numbers like -12 and 12 are opposites. Powers have opposites too, but the opposite may not always be obvious.

Example 1

What is the opposite of 2^3?

Solution

$2^3 = 8$ and $-2^3 = -8$

Thus, the opposite of 2^3 is -2^3.

Note: 2^3 is read as "two cubed"; -2^3 is read as "the opposite of two cubed"; and $(-2)^3$ is read as "negative two cubed." The parentheses in $(-2)^3$ are used to indicate that the negative sign is part of the base.

Example 2

What is the opposite of $(-5)^4$?

Solution

$(-5)^4 = 625$ and $-(-5)^4 = -625$

Thus, the opposite of $(-5)^4$ is $-(-5)^4$.

Also, $-5^4 = -(5 \times 5 \times 5 \times 5)$
$= -625$

Therefore, -5^4 is also the opposite of $(-5)^4$.

29. Evaluate $(-5)^4$ and -5^4 using a scientific calculator. Write out the keystrokes for each evaluation.

30. Are the answers in question 29 the same?

31. When -5^4 is evaluated using pencil and paper, the answer is -625.

a. What is the calculator doing when it evaluates -5^4?

b. How can you get the calculator to give the same answer as the paper-and-pencil method when evaluating -5^4?

It is important to note that $(-5)^4$ and -5^4 are not equivalent, even though the calculator gives the same answer for these two powers.

32. Indicate the sign that would result if you evaluated each of the following powers.

a. -8^2 **b.** $(-8)^3$ **c.** $(-8)^4$

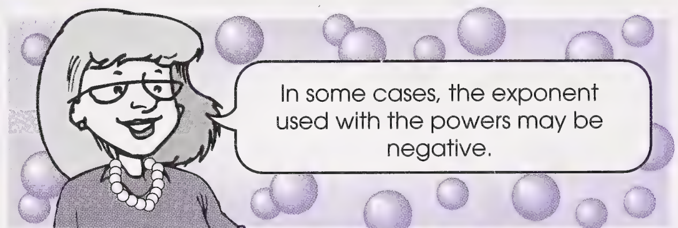
d. $-(-3)^2$ **e.** $-(-2)^5$

33. Evaluate the powers in question 32.

34. Indicate which is greater in each of the following pairs of powers.

a. $(-4)^2$ or -4^2 **b.** -3^4 or $-(-3)^4$

Check your answers by turning to the Appendix.



Example 3

What is the value of 6^{-2}?

Solution

$$6^{-2} = \left(\frac{1}{6}\right)^2 \text{ or } \boxed{6}\ \boxed{x^y}\ \boxed{2}\ \boxed{+/-}\ \boxed{=}$$

$$= \frac{1}{6} \times \frac{1}{6}$$

$$= \frac{1}{36}$$

0.027777777

The value of 6^{-2} is $\frac{1}{36}$ or 0.028, rounded to three decimal places.

Example 4

What is the value of $\left(\frac{2}{3}\right)^{-2}$?

Solution

$$\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 \text{ or } \boxed{2}\ \boxed{a\tfrac{b}{c}}\ \boxed{3}\ \boxed{x^y}\ \boxed{2}\ \boxed{+/-}\ \boxed{=}$$

$$= \frac{3}{2} \times \frac{3}{2}$$

$$= \frac{9}{4}$$

2.25

The value of $\left(\frac{2}{3}\right)^{-2}$ is $\frac{9}{4}$ or 2.25.

Example 5

What is the value of $(-0.45)^{-3}$? Round to two decimal places.

Solution

$\boxed{\cdot}\ \boxed{4}\ \boxed{5}\ \boxed{+/-}\ \boxed{x^y}\ \boxed{3}\ \boxed{+/-}\ \boxed{=}$

- 10.9739369

$$(-0.45)^{-3} = \frac{1}{(-0.45)^3}$$

The value of $(-0.45)^{-3}$, rounded to two decimal places, is -10.97.

35. Express each of the following as a positive exponent; then evaluate.

a. 10^{-3} **b.** 2^{-5} **c.** $\left(\frac{1}{4}\right)^{-2}$

d. 5^{-2} **e.** $(-3)^{-3}$ **f.** $(0.5)^{-4}$

36. Which symbol (>, <, or =) would you use to make each of the following a true statement? Change the form to make equivalent forms but do not evaluate.

a. $4^{-2} \bigcirc 5^{-2}$ **b.** $4^{-10} \bigcirc (-4)^{10}$

c. $8^{-10} \bigcirc 0$ **d.** $-11^{0} \bigcirc 11^{0}$

e. $3^{-5} \bigcirc \left(\frac{1}{3}\right)^{5}$ **f.** $4^{-9} \bigcirc (-4)^{9}$

37. How is a power with a negative exponent related to a power with a positive exponent?

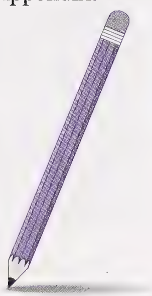
38. Evaluate each of the following as indicated.

a. $4^{-1} \times 2^{-1}$ **b.** $4^{-2} + 4^{0}$

c. $\left(\frac{1}{8}\right)^{-2} + \left(\frac{1}{10}\right)^{-2}$ **d.** $\left(\frac{4}{5}\right)^{-2} + \left(\frac{2}{3}\right)^{-4}$

Check your answers by turning to the Appendix.

Finding the Missing Exponent



You can put a number in exponential form by using division to find the exponent.

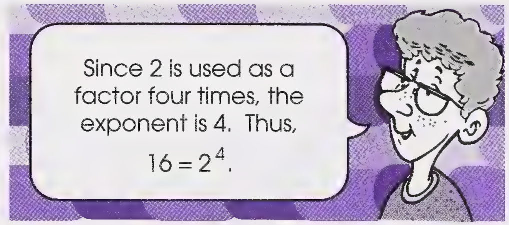
Using paper and pencil, keep dividing by 2 and see how many times 2 is a factor.

Example 1

Write 16 as a power of 2.

Solution

$$\begin{array}{r|l} 2 & 16 \\ 2 & 8 \\ 2 & 4 \\ 2 & 2 \\ & 1 \end{array}$$



39. Find the missing exponent using division.

a. $81 = 3^{\square}$ **b.** $32 = 2^{\square}$ **c.** $1331 = 11^{\square}$

Check your answers by turning to the Appendix.

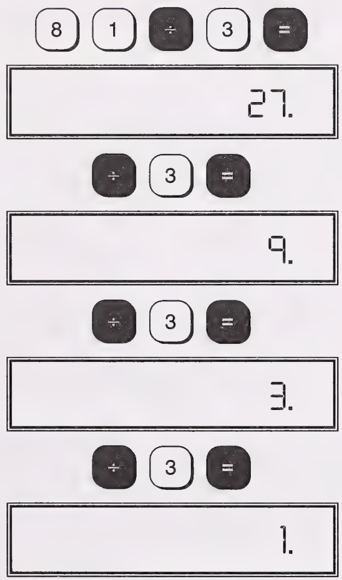
You can also use a scientific calculator to find a missing exponent. You can divide the number by the base until the number becomes 1 and keep track of how many times you divide; or you can use guess and test. With guess and test, various values for the exponent with the given base are tested using the power key.

Example 2

Write 81 as a power of 3.

Solution

Method 1: Using Division

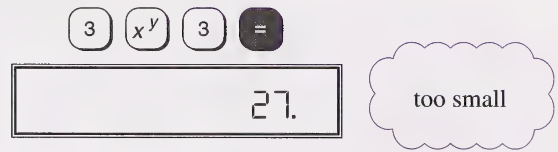


You had to divide 81 by 3 four times; thus, $81 = 3^4$.

Method 2: Using Guess and Test

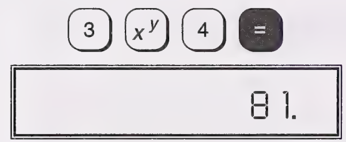
$3^{?} = 81$

Try the value of 3 for the exponent.



Try the value of 5 for the exponent.

Try the value of 4 for the exponent.



The exponent is 4 since $3^4 = 81$.

Use a scientific calculator to answer question 40.

40. Find the missing exponent in each of the following using division or guess and test.

a. $256 = 2^{\ }$ **b.** $3^{\ } = 6561$

c. $(1.4)^{\ } = 3.8416$ **d.** $-78\,125 = (-5)^{\ }$

Check your answers by turning to the Appendix.

Patterns in Powers

As in all aspects of mathematics, there are many patterns and relationships to explore in your work with powers. In powers, patterns can be helpful in determining an answer without solving.

Earlier in this activity you used patterns to help discover the value of a zero exponent and negative exponents.
Now you will use patterns to help you predict the last digit in the standard form of powers.

Example

Without evaluating, determine the last digit in the standard form of 2^{18}.

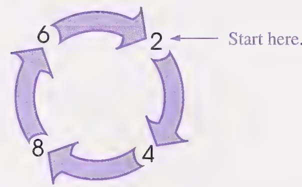
Solution

Use a chart like this to find a pattern.

Power Form	Standard Form	Last Digit
2^1	2	2
2^2	4	4
2^3	8	8
2^4	16	6
2^5	32	2
2^6	64	4
2^7	128	8
2^8	256	6

All the standard forms of the powers of 2 end in 2, 4, 8, or 6.

Moreover, the last digits repeat in a cyclic (or circular) pattern.



$$\left.\begin{aligned} &2^1 \text{ ends in } 2.\\ &2^2 \text{ ends in } 4.\\ &2^3 \text{ ends in } 8.\\ &2^4 \text{ ends in } 6.\end{aligned}\right\}\text{one cycle}\qquad \left.\begin{aligned} &2^5 \text{ ends in } 2.\\ &2^6 \text{ ends in } 4.\\ &2^7 \text{ ends in } 8.\\ &2^8 \text{ ends in } 6.\end{aligned}\right\}\text{cycle repeats}$$

This pattern will repeat as you move through the rest of the powers of 2.

You can apply the pattern to find the last digit of 2^{18}. You must go through four complete cycles and two steps into the fifth cycle.

Therefore, 2^{18} ends in 4.

41. Describe the pattern of the last digit in the standard form of powers with these bases.

a. 3 **b.** 4 **c.** 5 **d.** 7

42. Predict the last digit in the standard forms of these powers.

a. 2^{25} **b.** 3^{18} **c.** 7^{11} **d.** 5^{12}

43. Describe the pattern of the last digits in the standard forms of 2^5, 3^5, 4^5, 5^5, 6^5, 7^5, 8^5, 9^5, 10^5, 11^5,….

44. Predict the last digit in the standard forms of these powers.

a. 13^5 **b.** 32^5 **c.** 17^5

45. A mathematician by the name of Alfred Moesner discovered that he could produce the sequence 1, 4, 9, 16, 25,… by doing the following steps:

Step 1: Write the numbers 1, 2, 3,…. Extend the list to 10.

1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

Step 2: Cross out every second number.

1, $\cancel{2}$, 3, $\cancel{4}$, 5, $\cancel{6}$, 7, $\cancel{8}$, 9, $\cancel{10}$.

Step 3: Write 1 in the second row, and complete the second row by adding each number in the second row to the next number which is not crossed out in the first row.

1, $\cancel{2}$, 3, $\cancel{4}$, 5, $\cancel{6}$, 7, $\cancel{8}$, 9, $\cancel{10}$

1 4 9 16 25

a. Write each number in the second row as a square.

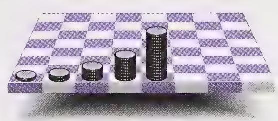
b. Predict the next three numbers in the sequence 1, 4, 9, 16, 25,….

Check your answers by turning to the Appendix.

Now Try This

Use a problem-solving strategy to answer the following questions.

46. Suppose you were to stack loonies on a chessboard in a way such that you have 1 loonie on the first square, 2 loonies on the second square, 4 loonies on the third square, 8 loonies on the fourth square, 16 loonies on the fifth square, and so on. What square will be the first to have over $1 000 000 on it? Explain how to arrive at this answer.

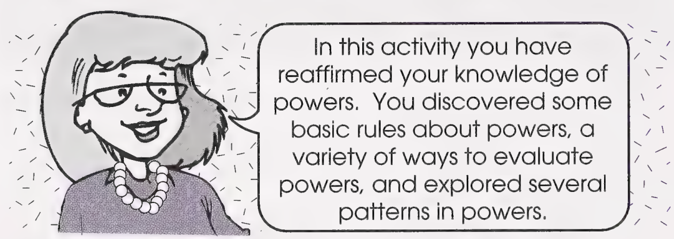


47. Twice a number is always smaller than the number squared. What is the smallest whole number for which this statement is always true? Show how you found this number.

Check your answers by turning to the Appendix.

Did You Know?

Knowledge about zero and negative exponents can be useful in tuning a piano. For instance, the A note above middle C on the keyboard (called concert A) has a frequency of 440×2^0. Other A notes as you go down the scale have frequencies of 440×2^{-1}, 440×2^{-2}, and so on.



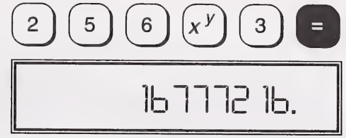
Activity 2: Multiplying and Dividing Powers

A very simple pattern is repeated many times to make the design on the right. Often, in your work in mathematics, you look for patterns that you can apply in order to perform computations. In this activity you will look for patterns in multiplying and dividing powers.

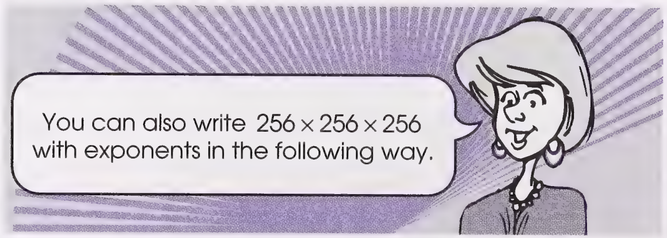
Multiplying Powers

In the section introduction you were told that a colour SVGA computer monitor can produce over 16 million colours using the three colours—red, blue, and green. You also found that the number of colours produced can be written as $256 \times 256 \times 256$ or 256^3.

Using a scientific calculator, you can determine exactly how many colours can be produced.



The SVGA colour monitor can display 16 777 216 different colours.

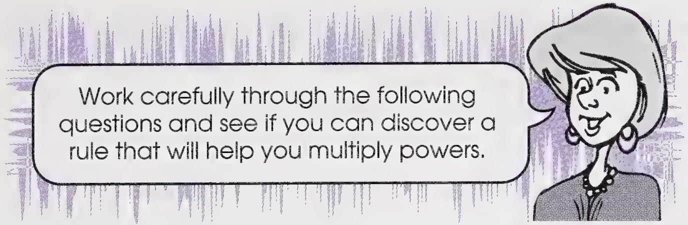


$$256 = 2^8$$
$$\therefore\ 256 \times 256 \times 256 = 2^8 \times 2^8 \times 2^8$$

You can find $2^8 \times 2^8 \times 2^8$ by multiplying powers.

$$\begin{aligned} 2^8 \times 2^8 \times 2^8 &= (2\times2\times2\times2\times2\times2\times2\times2) \\ &\quad \times(2\times2\times2\times2\times2\times2\times2\times2) \\ &\quad \times(2\times2\times2\times2\times2\times2\times2\times2) \\ &= 2^{24} \end{aligned}$$

Count the number of 2s.



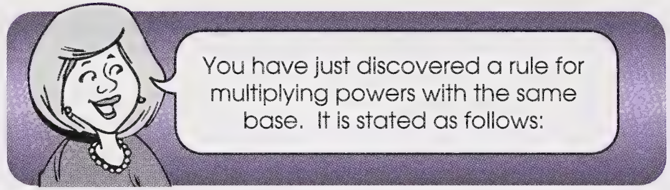
1. Copy and complete the following chart in your notebook. The first one is done as an example.

Expression	Factored Form	Power Form
$3^4 \times 3^3$	$(3\times3\times3\times3)\times(3\times3\times3)$ $=3\times3\times3\times3\times3\times3\times3$	3^7
$4^2 \times 4^3$		
5×5^3		
$2^2 \times 2^3 \times 2^4$		

2. What shortcut method do you see for multiplying powers?

3. Explain why the shortcut does not work for $3^2 \times 4^5$.

Check your answers by turning to the Appendix.



To multiply powers that have the same base, add the exponents while keeping the same base.

Example 1

What is $3^4 \times 3^5$?

Solution

$$\begin{aligned} 3^4 \times 3^5 &= 3^{4+5} \\ &= 3^9 \end{aligned}$$

Example 2

What is $(-4)^3 \times (-4)^5$?

Solution

$$\begin{aligned} (-4)^3 \times (-4)^5 &= (-4)^{3+5} \\ &= (-4)^8 \end{aligned}$$

4. Write each of the following as a single power where possible.

a. $2^3 \times 2^7$

b. $(-5)^8 \times (-5)^{10}$

c. $\left(\frac{1}{2}\right)^5 \times \left(\frac{1}{2}\right)^6$

d. $(0.5)^3 \times (2.5)^4$

e. $8^3 \times 8^4 \times 8^5$

f. $3^2 \times 3^9 \times 5 \times 5^4$

Check your answers by turning to the Appendix.

The exponents used in multiplying powers can be negative.

Example 3

Find the value of $3^{-2} \times 2^{-1}$.

Solution

$$
\begin{aligned}
3^{-2} \times 2^{-1} &= \left(\frac{1}{3}\right)^2 \times \frac{1}{2} \\
&= \frac{1}{9} \times \frac{1}{2} \\
&= \frac{1}{18}
\end{aligned}
$$

To find the answer to Example 3 using a calculator, enter the following keystrokes.

3 x^y 2 +/− ×

0.111111111

2 x^y 1 +/− =

0.055555555

The value of $3^{-2} \times 2^{-1}$ is 0.056, rounded to three decimal places.

Is $\frac{1}{18}$ equal to 0.055…? Check using a scientific calculator.

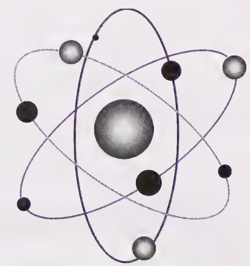
5. Express the following as a single power; then find the value.

a. $8^2 \times 8^{-3}$

b. $2^{-3} \times 2^{-2}$

c. $\left(\frac{1}{4}\right) \times \left(\frac{1}{4}\right)^{-3}$

6. In an atom, an electron makes about 10^{15} orbits per second circling the nucleus. How many orbits does it make in the following times.



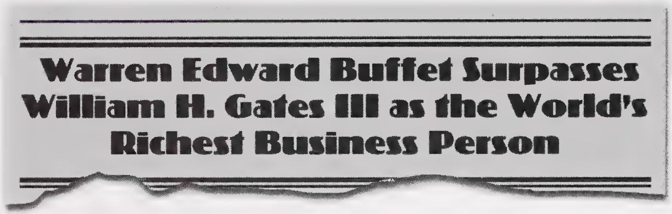
a. 10^4 s

b. 10^{11} s

7. Dale has a stereo system with an amplifier as well as a preamplifier. The preamplifier boosts the signal by a factor of 10^5, and the amplifier boosts the signal by a factor of 10^4. Calculate the factor by which the signal is boosted after it has passed through both amplifiers if this factor is a product of the two individual factors.

Check your answers by turning to the Appendix.

Dividing Powers


Warren Edward Buffet Surpasses William H. Gates III as the World's Richest Business Person

As of October 3, 1996, Warren Edward Buffet, CEO of Berkshire Hathaway (a stock investment company), has become the richest business person in the world with an amassed fortune of 16.6 billion dollars. The fortune of William H. Gates III at the same time was 14.1 billion dollars. **Note:** Due to the rise and fall of the stock market this could change at any time.

You can write the fortunes of Warren Edward Buffet and William H. Gates III in power form as 1.66×10^{10} and 1.41×10^{10}. You will study writing numbers in this form in more detail in Section 2.

Suppose you were to spend \$1 million dollars a year. How long would it take you to spend each of the fortunes stated?

To answer this question you would have to divide powers as follows:

$$\begin{aligned}\frac{1.66 \times 10^{10}}{10^6} &= 1.66 \times \frac{10^{10}}{10^6} \\ &= 1.66 \times 10^{10-6} \\ &= 1.66 \times 10^4 \\ &= 16\,600\end{aligned}$$

It would take 16 600 years to spend the fortune of Warren Edward Buffet at a rate of 1 million dollars per year.

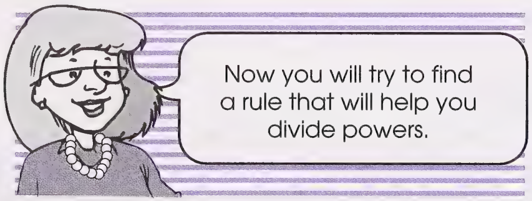
Also,
$$\begin{aligned}\frac{1.41 \times 10^{10}}{10^6} &= 1.41 \times \frac{10^{10}}{10^6} \\ &= 1.41 \times 10^{10-6} \\ &= 1.41 \times 10^4 \\ &= 14\,100\end{aligned}$$

It would take 14 100 years to spend the fortune of William H. Gates at a rate of 1 million dollars per year.

8. Would you be able to spend either fortune in a lifetime at this rate?

Check your answers by turning to the Appendix.

You may wish to use the Internet to search for more information on the richest people in the world. Enter the words “richest people” on any of the Internet’s search engines.



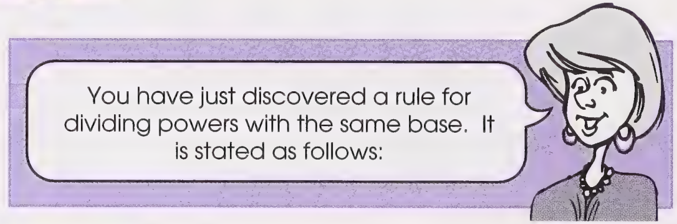
9. Copy and complete the following table in your notebook. The first one is done as an example.

Expression	Factored Form	Power Form
$5^6 \div 5^2$	$(5\times5\times5\times5\times5\times5)\div(5\times5)$ $=\dfrac{\overset{1}{\cancel{5}}\times\overset{1}{\cancel{5}}\times5\times5\times5\times5}{\underset{1}{\cancel{5}}\times\underset{1}{\cancel{5}}}$ $=5\times5\times5\times5$	5^4
$2^5 \div 2^3$		
$4^6 \div 4^5$		
$3^7 \div 3^4$		

10. What shortcut method do you see for dividing powers?

11. Explain why the rule does not work for $5^6 \div 2^3$.

Check your answers by turning to the Appendix.



To divide powers that have the same base, subtract the exponents while keeping the same base.

Example 1

Write $\frac{8^5}{8^2}$ as a single power.

Solution

$$\frac{8^5}{8^2}=8^{5-2}$$
$$=8^3$$

Example 2

Write $(-3)^4 \div (-3)^2$ as a single power.

Solution

$$\begin{aligned}(-3)^4 \div (-3)^2 &= (-3)^{4-2} \\ &= (-3)^2\end{aligned}$$

12. Write each of the following as a single power where possible. If it is not possible, explain why.

 a. $3^6 \div 3^2$
 b. $(-5)^7 \div (-5)^3$
 c. $(0.8)^9 \div (0.8)^8$
 d. $\dfrac{5^7}{5^7}$
 e. $\dfrac{7^{12}}{12^7}$
 f. $\left(\dfrac{1}{2}\right)^7 \div \left(\dfrac{1}{2}\right)^2$

Check your answers by turning to the Appendix.

Sometimes dividing powers involves negative exponents.

Example 3

Write $5^{-4} \div 5^{-2}$ as a single power.

Solution

$$\begin{aligned}5^{-4} \div 5^{-2} &= 5^{-4-(-2)} \\ &= 5^{-4+2} \\ &= 5^{-2}\end{aligned}$$

13. Write each of the following as a single power.

 a. $5^2 \div 5^4$
 b. $(-3)^2 \div (-3)^5$
 c. $4^{-2} \div 4^{-4}$

Check your answers by turning to the Appendix.

Now Try This

Try the following challenging questions.

14. Simplify each of the following.

 a. $\dfrac{8^2 \times 8^3}{8^4}$
 b. $\dfrac{9^2 \times 9^3 \times 9^4}{9 \times 9^5}$
 c. $\left(4^2 \times 4^6 \times 4\right) \div \left(4^5 \times 4 \times 4\right)$

15. The mass of the Sun is about 10^{27} t, and the mass of Earth is about 10^{22} t. How many times greater is the mass of the Sun than the mass of Earth?

Check your answers by turning to the Appendix.

Non-Numeric Bases

In Activity 1 you used powers to represent the number of amoebas after each division. You could have used a letter such as n to represent the number of divisions.

Number of Divisions	Number of Amoebas
1	2^1
2	2^2
3	2^3
4	2^4
5	2^5
•	•
•	•
•	•
n	2^n

Since the exponent in the power used to represent the number of amoebas is the number of divisions, you can write a general form for the number of amoebas as 2^n.

Similarly, you can use a variable as a base. For example, you could have n^3 as a power. What does n^3 mean?

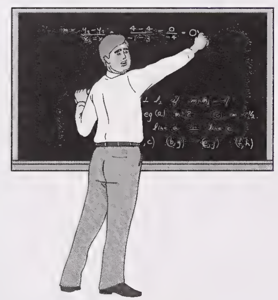
$$2^3 = 2 \times 2 \times 2$$
$$\therefore\ n^3 = n \times n \times n$$

The exponent shows you how many times the base is used as a factor.

You can also have variables for both the base and the exponent. What does n^a mean? In words, n^a means that the factor n is used as a factor a times. You can write this as follows:

$$n^a = \underbrace{n \times n \times n \times \ldots \times n}_{a \text{ factors}}$$

In this activity, you discovered some properties or rules for multiplication and division of powers using patterns and specific numbers. Now you are going to generalize those rules by using variables to replace the numbers.



Example 1

What does $n^3 \times n^2$ mean?

Solution

$$n^3 \times n^2 = (n \times n \times n) \times (n \times n) \quad \text{or} \quad n^3 \times n^2 = n^{3+2}$$
$$= n^5 \qquad\qquad = n^5$$

Example 2

What is $n^a \times n^b$ in simplest form?

Solution

$$n^a \times n^b = \underbrace{(n \times n \times n \times \ldots \times n)}_{a \text{ factors}} \times \underbrace{(n \times n \times n \times \ldots \times n)}_{b \text{ factors}}$$
$$= n^{a+b}$$

As you can see, the rule is the same as discovered previously, except variables have been used to represent all the possible number replacements.

When multiplying, keep the base the same and add the exponents.

$$n^a \times n^b = n^{a+b}$$

Similarly, to divide powers, keep the base the same and subtract the exponents. Remember division by zero is undefined; therefore, $n \neq 0$.

$$n^a \div n^b = n^{a-b}$$

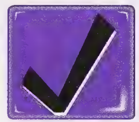
$n \neq 0$ and a and b are integers.

16. Simplify the following:

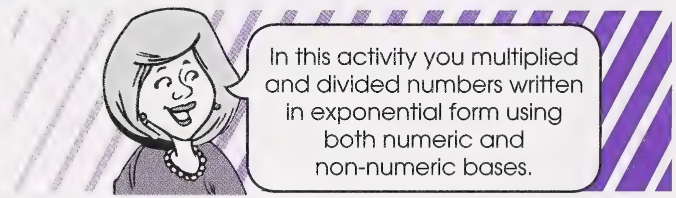
a. $p \times p \times p \times p \times p \times p \times p$ **b.** $m^2 \times m^4$

c. $n \times n^2 \times n^3$ **d.** $p^8 \div p^3$

e. $\dfrac{a^6}{a^3}$



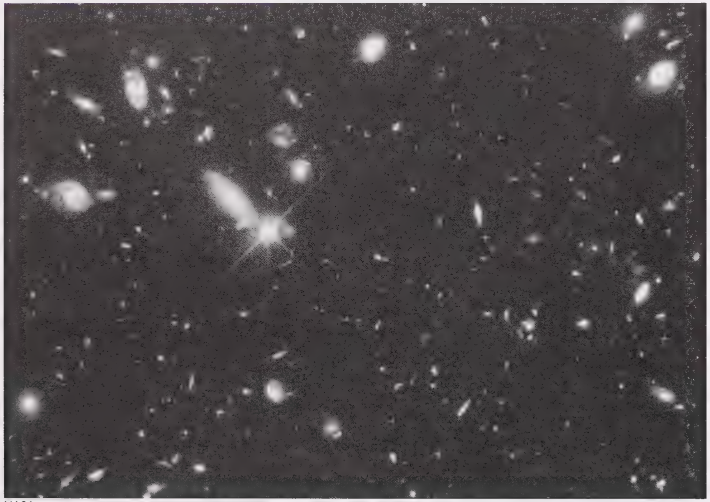
Check your answers by turning to the Appendix.



Activity 3: Power Rules

There are several rules that involve more than one power. These rules have extensive application in the area of science. In this activity you will discover the rules for the power of a power, the power of a product, and the power of a quotient. You will then extend the rules to variable bases.

Power of a Power



NASA

Hubble Space Telescope Deep Field View of Hundreds of Galaxies

It has been estimated that each galaxy contains 10^{11} stars. If there are about 10^{11} galaxies in the universe, approximately how many stars are there in total? You can express the numbers of stars as $10^{11} \times 10^{11}$ or $\left(10^{11}\right)^2$.

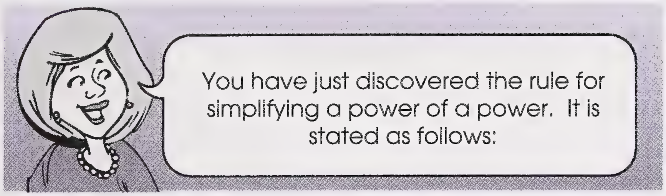
What is $\left(10^{11}\right)^2$ equal to as a single power? As you do the following question, see if you can discover a rule that tells you how to simplify a power of a power.

1. Copy and complete the following table in your notebook. The first one is done as an example.

Expression	Factored Form	Power Form
$\left(5^3\right)^2$	$5^3 \times 5^3$ $= (5\times5\times5)\times(5\times5\times5)$ $= 5\times5\times5\times5\times5\times5$	5^6
$\left(7^2\right)^3$		
$\left(3^4\right)^2$		
$\left(2^3\right)^4$		

2. What shortcut method do you see for simplifying the power of a power?

Check your answers by turning to the Appendix.



To simplify a power of a power, multiply the exponents and keep the same base.

Example 1

What is $\left(10^2\right)^3$ in simplest power form?

Solution

$$\begin{aligned}\left(10^2\right)^3 &= 10^{2\times 3}\\ &= 10^6\end{aligned}$$

Example 2

What is $\left[(-5)^4\right]^2$ in simplest power form?

Solution

$$\begin{aligned}\left[(-5)^4\right]^2 &= (-5)^{4\times 2}\\ &= (-5)^8\end{aligned}$$

Example 3

Express $\left[(-2)^2\right]^{-3}$ in simplest power form.

Solution

$$\begin{aligned}\left[(-2)^2\right]^{-3} &= (-2)^{2\times(-3)}\\ &= (-2)^{-6}\end{aligned}$$

3. Write each of the following as a single, power with a positive exponent.

   a. $\left(3^5\right)^4$ b. $\left[(-2)^3\right]^7$

   c. $\left[(0.2)^5\right]^3$ d. $\left[\left(\frac{5}{8}\right)^2\right]^6$

e. $\left(4^{2}\right)^{-1}$

f. $\left(10^{-2}\right)^{-3}$

4. You may recall from previous mathematics courses that the terms googol and googolplex are used for very large numbers.

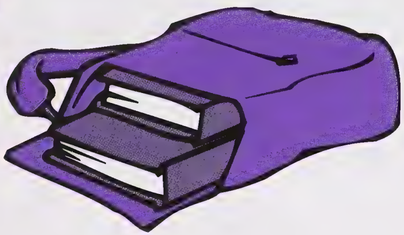
   a. A googol is written as $\left(10^{10}\right)^{10}$. Write this number as a power with a single exponent.

   b. A googolplex is 10^{googol} or $10^{\left(10^{10}\right)^{10}}$. Describe in words what the exponent would be for a googolplex written as a power with a single exponent.

5. Can you evaluate either of the numbers in question 4 with a scientific calculator? What is the largest number you can enter?

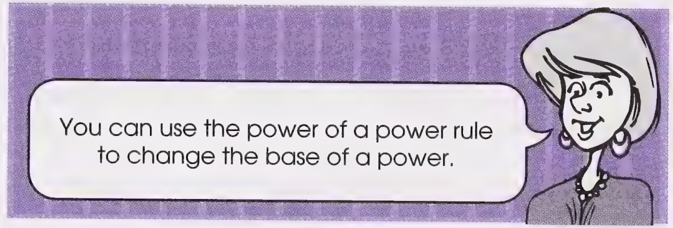
6. Explain why $\left(3^{4}\right)^{2}=\left(3^{2}\right)^{4}$ by writing each as a product of equal factors.

Check your answers by turning to the Appendix.



Changing Bases

Sometimes it is useful to be able to change the base of a power. For example, being able to change bases can allow you to decide which of two powers is greater without evaluating the powers.



Example 1

Write 8^{2} as a power with a base of 2.

Solution

$$\begin{aligned} 8^{2} &= (2\times 2\times 2)^{2} \\ &= \left(2^{3}\right)^{2} \\ &= 2^{3\times 2} \\ &= 2^{6} \end{aligned}$$

Example 2

Write 3^8 as a power with a base of 9.

Solution

$$3^8 = 3^{2\times4}$$
$$= \left(3^2\right)^4$$
$$= 9^4$$

7. Complete each of the following.

a. $4^9 = 2^{\ }$ **b.** $16^2 = 2^{\ }$ **c.** $3^{12} = 27^{\ }$

Now Try This

Try these challenge questions.

8. Without evaluating, determine which power is greater in each pair.

a. 2^{20} or 4^9

b. 3^7 or 27^2

c. 16^5 or 2^{20}

9. Without calculating, write the following powers in order from smallest to largest.

2^{55} 3^{44} 4^{33} 5^{22}

Check your answers by turning to the Appendix.

You may wish to use the Internet to explore large powers of two. The following uniform resource locator (URL) will give you the Fun With Numbers home page. There you can locate information on powers of two.

http://www.mind.net/xethyr/numbers/index.html

Power of a Product

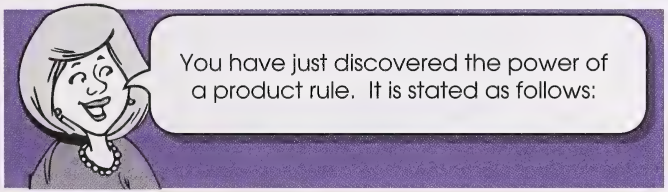
The expression $\left(8^2 \times 2^3\right)^5$ is called a power of a product. Is there a shorter way of writing $\left(8^2 \times 2^3\right)^5$? Your answers to the following question should help you discover a rule for simplifying a power of a product.

10. Copy and complete the following table in your notebook. The first one is done as an example.

Expression	Factored Form	Power Form
$(6^2 \times 4^3)^3$	$(6^2 \times 4^3) \times (6^2 \times 4^3) \times (6^2 \times 4^3)$ $= 6^2 \times 6^2 \times 6^2 \times 4^3 \times 4^3 \times 4^3$	$6^6 \times 4^9$
$(5^4 \times 2^7)^2$		
$(8 \times 7^4)^3$		
$(3^5 \times 4^6)^2$		

11. What shortcut method do you see for simplifying the power of a product?

Check your answers by turning to the Appendix.



To simplify a power of a product, multiply each of the exponents inside the brackets with the exponent outside the brackets.

Example 1

Write $(8^5 \times 3^4)^3$ in simplest power form.

Solution

$$\begin{aligned}(8^5 \times 3^4)^3 &= 8^{5\times3} \times 3^{4\times3}\\ &= 8^{15} \times 3^{12}\end{aligned}$$

Example 2

Write $\left[(-4)^2 \times \left(\frac{1}{2}\right)^3\right]^5$ in simplest power form?

Solution

$$\begin{aligned}\left[(-4)^2 \times \left(\frac{1}{2}\right)^3\right]^5 &= (-4)^{2\times5} \times \left(\frac{1}{2}\right)^{3\times5}\\ &= (-4)^{10} \times \left(\frac{1}{2}\right)^{15}\end{aligned}$$

Example 3

Write $\left(3^{-4} \times 4^{-2}\right)^{-3}$ in simplest form.

Solution

$$\left(3^{-4} \times 4^{-2}\right)^{-3} = 3^{(-4)\times(-3)} \times 4^{(-2)\times(-3)}$$
$$= 3^{12} \times 4^{6}$$

12. Express each of the following as a product of powers with positive exponents.

a. $\left(8^{7} \times 6^{2}\right)^{4}$

b. $\left[(-3)^{2} \times (-4)^{7}\right]^{3}$

c. $\left(7 \times 4^{2}\right)^{5}$

d. $\left[(0.5)^{4} \times (1.4)^{3}\right]^{2}$

e. $\left(5^{-6} \times 3^{7}\right)^{-2}$

f. $\left[(-3)^{2} \times \left(\frac{1}{3}\right)^{2}\right]^{-3}$

Use a scientific calculator to answer questions 13 and 14.

13. Evaluate the following products of powers. Follow the rules for order of operations. Round your answers to two decimal places where required.

a. $\left(6^{2} \times 3^{4}\right)^{2}$

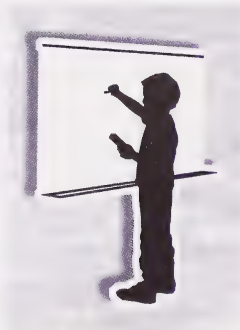
b. $\left[(0.3)^{2} \times (2.6)^{3}\right]^{-2}$

14. Evaluate question 12.d. as given in the question and the answer for question 12.d. as given in the Appendix. Do you get the same answer in each evaluation?

Check your answers by turning to the Appendix.

Power of a Quotient

Now that you have studied the power of a product, what about the power of a quotient? Is there a similarity? Check it out as you work through the following questions.

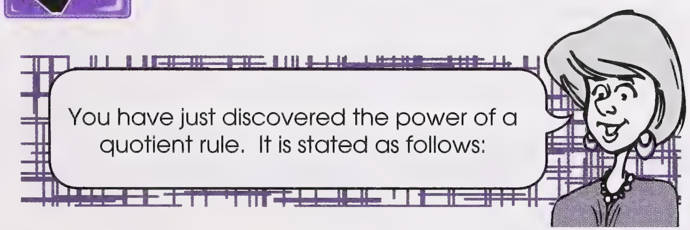


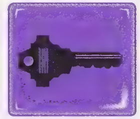
15. Copy and complete the following table in your notebook. The first one is done as an example.

Expression	Factored Form	Power Form
$\left(\frac{8^2}{4^3}\right)^3$	$\left(\frac{8^2}{4^3}\right)\times\left(\frac{8^2}{4^3}\right)\times\left(\frac{8^2}{4^3}\right) = \frac{8^2\times8^2\times8^2}{4^3\times4^3\times4^3}$	$\frac{8^6}{4^9}$
$\left(\frac{5^4}{3^2}\right)^2$		
$\left(\frac{4^7}{2^5}\right)^3$		

16. What shortcut method do you see for simplifying the power of a quotient?

Check your answers by turning to the Appendix.





To simplify the power of a quotient, multiply each of the exponents inside the brackets by the exponent outside the brackets.

Example 1

Write $\left(\frac{15^4}{8^5}\right)^3$ in simplest power form?

Solution

$$\left(\frac{15^4}{8^5}\right)^3 = \frac{15^{4\times3}}{8^{5\times3}} = \frac{15^{12}}{8^{15}}$$

Example 2

Write $\left[\frac{(-4)^3}{-6}\right]^4$ in simplest power form.

Solution

$$\left[\frac{(-4)^3}{-6}\right]^4 = \frac{(-4)^{3\times4}}{(-6)^{1\times4}} = \frac{(-4)^{12}}{(-6)^4}$$

Example 3

Write $\left[\frac{(-3)^{-2}}{(-3)^{-4}}\right]^5$ in simplest power form.

Solution

$$\begin{aligned}\left[\frac{(-3)^{-2}}{(-3)^{-4}}\right]^5 &= \frac{(-3)^{(-2)\times 5}}{(-3)^{(-4)\times 5}} \\ &= \frac{(-3)^{-10}}{(-3)^{-20}} \\ &= (-3)^{-10-(-20)} \\ &= (-3)^{-10+20} \\ &= (-3)^{10}\end{aligned}$$

17. Express each of the following as a quotient of powers.

a. $\left(\frac{8^4}{2^3}\right)^3$

b. $\left(\frac{7^4}{3^5}\right)^6$

c. $\left(\frac{4^7}{3^4}\right)^{-3}$

d. $\left[\frac{(-5)^{-2}}{(-5)^{-6}}\right]^{-2}$

18. Use a calculator to evaluate question 17.b. and to evaluate the answer to question 17.b. as given in the Appendix. Do you get the same answer in each evaluation? What does this indicate about the two expressions?

Check your answers by turning to the Appendix.

Non-Numerical Bases

In this activity, you discovered some additional properties or rules for powers using patterns and specific numbers. Now you are going to generalize those rules by using variables to replace the numbers.

Example 1

Express $\left(n^2\right)^3$, where n is any rational number, in simplest power form.

Solution

$$\begin{aligned}\left(n^2\right)^3 &= n^2 \times n^2 \times n^2 \\ &= n^{2+2+2} \\ &= n^6\end{aligned}$$

Example 2

Express $\left(n^{a}\right)^{b}$, where n is any rational number and a and b are integers, in simplest power form.

Solution

$$\left(n^{a}\right)^{b} = \underbrace{n^{a} \times n^{a} \times \ldots \times n^{a}}_{b \text{ factors}}$$

$$= n^{a \times b}$$

As you can see, the rule is the same as discovered previously, except variables have been used to represent all the possible number replacements.

To simplify a power of a power, multiply the exponents.

$$\left(n^{a}\right)^{b} = n^{ab}$$

To simplify a power of a product or a power of a quotient, multiply each of the exponents inside the brackets by the exponent outside the brackets.

$$\left(n^{a} \times m^{b}\right)^{c} = n^{ac} \times m^{bc}$$

$$\left(n^{a} \div m^{b}\right)^{c} = n^{ac} \div m^{bc}$$

19. Simplify the following.

a. $\left(a^{2}\right)^{3}$ **b.** $\left(y^{3}\right)^{4}$ **c.** $\left(m^{4} \times n^{3}\right)^{2}$

d. $\left(a^{2}b^{2}\right)^{3}$ **e.** $\left(\frac{c^{2}}{b^{3}}\right)^{2}$ **f.** $\left(a^{4} \div b^{2}\right)^{3}$

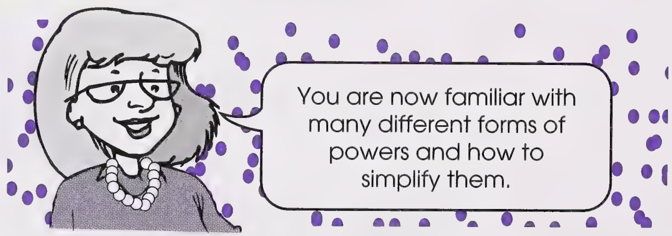
20. Use the exponent laws and guess and test to find an answer for n in the following.

a. $n^{8} \div n^{4} = 81$ **b.** $\left(n^{3}\right)^{2} = 729$

c. $n^{3} \times n^{2} = 32$ **d.** $\frac{n^{5} \times n^{2}}{n^{4}} = 125$

21. **a.** Find a value of n for which $n^{4} \times n^{5} = n^{9}$.
b. Can n be any value?
c. Which law does question 21.a. illustrate?

Check your answers by turning to the Appendix.



Now Try This

Use a problem-solving strategy to answer the following question.

22. **a.** Complete the following. Can you see a pattern in the answers? **Hint:** Express them as squares.

$1^3 = 1$

$1^3 + 2^3 = 9$

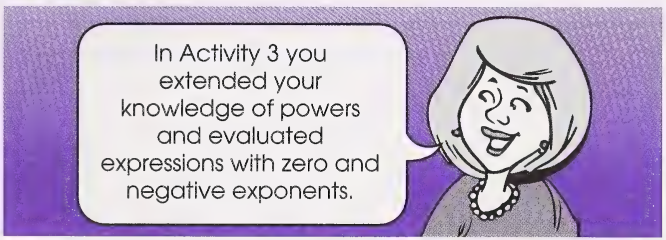
$1^3 + 2^3 + 3^3 =$

$1^3 + 2^3 + 3^3 + 4^3 =$

$1^3 + 2^3 + 3^3 + 4^3 + 5^3 =$

b. What would the sum of the first six cubic numbers be?

Check your answers by turning to the Appendix.



Activity 4: Order of Operations

You were shown in previous mathematics courses and in Module 1 that the order in which you do certain operations is very important. Just like many other activities that you may be involved with, doing the steps in a different order will produce a very different result. Just imagine the results if a person attempted to do the following steps in random order.

- Change a flat tire on a car.
- Get dressed in the morning.
- Start the car.
- Drive to work.

Mathematicians have agreed upon rules to follow in performing a variety of operations so that you can be consistent in your outcomes.

- Perform operations in **b**rackets first.
- Evaluate **e**xponents next.
- Then do **d**ivision and **m**ultiplication in order from left to right.
- Finally, do **a**ddition and **s**ubtraction in order from left to right.

Remember BEDMAS. This mnemonic will help you recall the order of operations.

B → **b**rackets
E → **e**xponents
D/M → **d**ivision/**m**ultiplication
A/S → **a**ddition/**s**ubtraction

Example 1

Evaluate $18-3^2 \times 4$.

Solution

$$\begin{aligned} & 18-3^2 \times 4 && \leftarrow \text{Evaluate the power.} \\ &= 18-9\times 4 && \leftarrow \text{Multiply.} \\ &= 18-36 && \leftarrow \text{Change subtraction to addition.} \\ &= 18+(-36) && \leftarrow \text{Add.} \\ &= -18 \end{aligned}$$

1. Why do you evaluate the power first in Example 1?

Check your answers by turning to the Appendix.

When using a scientific calculator to evaluate questions involving several operations, you can evaluate from left to right if you use brackets to have certain operations done first. Most scientific calculators follow the BEDMAS rule for order of operations; so, if you want a lower order operation performed first, then use brackets.

Example 2

Evaluate $18-3^2 \times 4$ using a calculator.

Solution

Perform the keystrokes in the order shown.

1 8 − 3 x^y 2 × 4 =

- 18.

2. Do the keystrokes in Example 2 in the order shown.

a. What happened when you pressed × ?

b. Why are no brackets required after − is pressed?

c. What function does = perform?

Check your answers by turning to the Appendix.

Example 3

Evaluate $\left(\frac{1}{2}\right)^2 + \frac{1}{4} \times 2$.

Solution

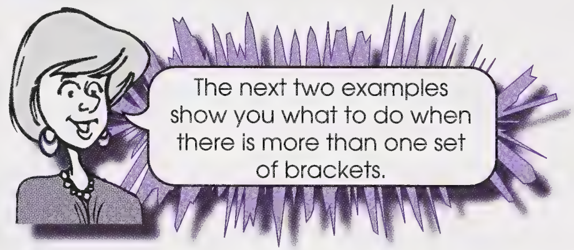
$$\begin{aligned}
&\boxed{\left(\frac{1}{2}\right)^2} + \frac{1}{4}\times 2 && \leftarrow \text{Evaluate the power.}\\
&= \frac{1}{4} + \boxed{\frac{1}{\cancel{4}_2}\times \cancel{2}^1} && \leftarrow \text{Multiply.}\\
&= \boxed{\frac{1}{4} + \frac{1}{2}} && \leftarrow \text{Find the common denominator.}\\
&= \boxed{\frac{1}{4} + \frac{2}{4}} && \leftarrow \text{Add.}\\
&= \frac{3}{4}
\end{aligned}$$

$$\frac{1}{2} \xrightarrow{\times 2} = \xrightarrow{\times 2} \frac{2}{4}$$

3. Evaluate Example 3 using a scientific calculator. **Note:** If you have a fraction key ($a\frac{b}{c}$), then you should be able to enter the operations in order. If you do not have a fraction key, then you will have to use brackets or the equal key to have $\frac{1}{2}$ changed to a decimal before using the exponent key.

4. Which method uses fewer keystrokes?

Check your answers by turning to the Appendix.



Example 4

Evaluate $10^2 - \left[3^2 \times (4+1)\right]$.

Solution

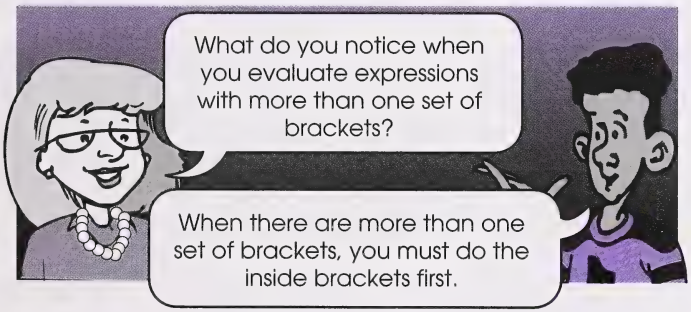
$$\begin{aligned}
&10^2 - \left[3^2 \times \boxed{(4+1)}\right] && \leftarrow \text{Work in round brackets first; add.}\\
&= 10^2 - \left[\boxed{3^2} \times 5\right] && \leftarrow \text{Evaluate the power in the square brackets.}\\
&= 10^2 - \boxed{[9\times 5]} && \leftarrow \text{Work in square brackets; multiply.}\\
&= \boxed{10^2} - 45 && \leftarrow \text{Evaluate the power.}\\
&= \boxed{100-45} && \leftarrow \text{Subtract.}\\
&= 55
\end{aligned}$$

Example 5

Evaluate $6 \div \left[3^2 - (-2+1)\right]$.

Solution

$$
\begin{aligned}
& 6 \div \left[3^2 - (-2+1)\right] && \leftarrow \text{Work in round brackets first; add.}\\
&= 6 \div \left[3^2 - (-1)\right] && \leftarrow \text{Evaluate the power in the square brackets.}\\
&= 6 \div \left[9 - (-1)\right] && \leftarrow \text{Change subtraction to addition.}\\
&= 6 \div [9+1] && \leftarrow \text{Work in square brackets; add.}\\
&= 6 \div 10 && \leftarrow \text{Divide.}\\
&= \frac{6}{10} && \leftarrow \text{Simplify.}\\
&= \frac{3}{5}
\end{aligned}
$$



5. Now evaluate Example 4 and Example 5 using a scientific calculator.

6. Did you find that you needed two sets of brackets in Example 5 while in Example 4 you needed only one set of brackets? Why?

Check your answers by turning to the Appendix.

Sometimes there will be a power of a mathematical expression.

To evaluate such an expression, work inside the brackets first and follow the rules of BEDMAS (as in the previous examples). Apply the exponent outside the brackets as the final step in the solution.

Example 6

Evaluate $\left[3^2 + 5 \times (-1)^3\right]^{-2}$.

Solution

$$
\begin{aligned}
& \left[3^2 + 5 \times (-1)^3\right]^{-2} && \leftarrow \text{Work in square brackets first; evaluate the powers.}\\
&= \left[9 + 5 \times (-1)\right]^{-2} && \leftarrow \text{Multiply.}\\
&= \left[9 + (-5)\right]^{-2} && \leftarrow \text{Add.}\\
&= 4^{-2} && \leftarrow \text{Evaluate the power.}\\
&= \frac{1}{16}
\end{aligned}
$$

7. Evaluate Example 6 using a scientific calculator.

8. Did you find that any brackets were needed in question 7?

9. Perform the following operations.

a. $3^3 - 3\times 2$

b. $2^3 - 2^2$

c. $\dfrac{4^3}{4^2 \times 2}$

d. $\dfrac{9^2 - (3\times 6)}{12-(4+1)}$

e. $\dfrac{(12-6)\times(12+6)}{(3+3)^2}$

f. $\dfrac{3^2\times 4+3\times 2}{16\div 4+8+2}$

g. $2^2+3^2+4^2-(2+3)^2$

Use a scientific calculator to answer questions 10 and 11.

10. Perform the following operations.

a. $(-8)^2 - 5^2$

b. $\left[\dfrac{(-3)^2+(-8)}{-2}\right]^{-3}$

c. $-2\left[(-2)^2+(-3)^2+(-4)^2\right]$

d. $\dfrac{2\times(-3)\times(-12)}{-16\div(-2)^4}$

11. Do question 10.c. in a more efficient way than shown in the answer to 10.c. in the Appendix.

12. Place parentheses in each equation to make it true.

a. $3+6^2 \div 14-1=3$

b. $27+7.8\div 2-4=13.4$

c. $4+5^2+3^2\div 10=9$

d. $\dfrac{3\times 4^2+2}{34-3+4}=2$

13. Identify the error in order of operations for each of these calculations.

a. $4\times 7+6\times 2=104$

b. $7+2\times 3^2=43$

c. $6+2^3\times 3-12\div 3=10$

Check your answers by turning to the Appendix.

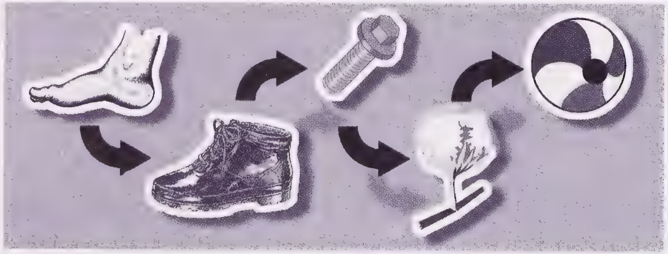
Now Try This

You are now ready for a problem-solving excursion.

You've probably played the word game called **doublets**. It's a game invented by Lewis Carroll where the object is to change one word into another word with the same number of letters in as few steps as possible. Each step involves changing one letter at a time so that each new combination also forms a real word.

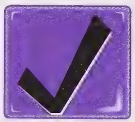
For example, to change FOOT to BALL, you would make the following changes.

FOOT → BOOT → BOLT → BOLL → BALL

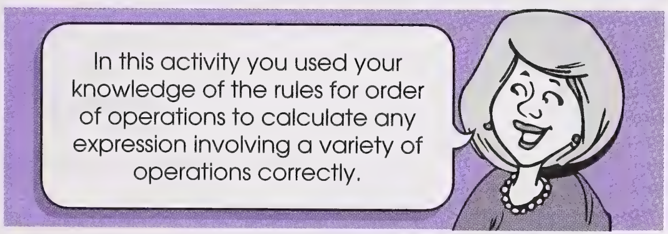


Changing FOOT to BALL was done in the minimum number of steps.

14. Write out the steps required to change MATH to TOPS. Try to use as few steps as possible.



Check your answers by turning to the Appendix.



Activity 5: Simplifying Expressions

In the previous activities you examined the various exponent rules. In this activity you will discover how to use these rules to simplify expressions with variable bases.

The following are examples of algebraic expressions.

- $\dfrac{44x^5y^4}{22x^2y^5}$
- $\dfrac{12ab^2}{3a^{-2}b^4}$

Expressions such as these can often be written in a simpler form.

Example 1

Simplify $\dfrac{44x^5y^4}{22x^2y^5}$.

Solution

$$\begin{aligned}\frac{44x^5y^4}{22x^2y^5} &= \frac{44}{22}\times\frac{x^5}{x^2}\times\frac{y^4}{y^5}\\ &= 2x^{5-2}y^{4-5}\\ &= 2x^3y^{-1}\\ &= \frac{2x^3}{y^1} \text{ or } \frac{2x^3}{y}\end{aligned}$$

Example 2

Simplify $\dfrac{12ab^2}{3a^{-2}b^4}$.

Solution

$$\begin{aligned}\frac{12ab^2}{3a^{-2}b^4} &= \frac{12}{3} \times \frac{a^1}{a^{-2}} \times \frac{b^2}{b^4} \\ &= 4a^{1-(-2)}b^{2-4} \\ &= 4a^{1+2}b^{-2} \\ &= \frac{4a^3}{b^2}\end{aligned}$$

Example 3

Simplify $2x^2z\left(3xy^2\right)$.

Solution

$$\begin{aligned}2x^2z\left(3xy^2\right) &= 2 \times 3 \times x^{2+1} \times y^2 \times z \\ &= 6x^3y^2z\end{aligned}$$

1. Study the preceding examples, and write a general rule for simplifying algebraic expressions.

2. Simplify the following algebraic expressions.

   a. $\dfrac{10x^2y^3}{2xy^2}$

   b. $\dfrac{18x^3y}{6x^3}$

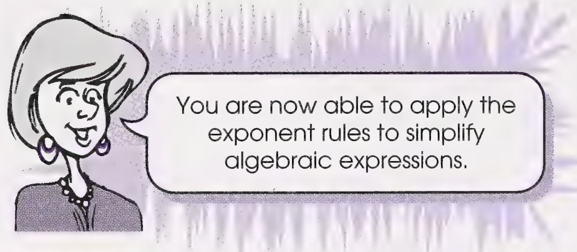
   c. $\dfrac{14a^{-5}b^2}{7a^{-1}b^{-4}}$

   d. $n^2p^3\left(2n^{-1}p^2\right)$

   e. $6a^5b^{-2}\left(3ac^3\right)$

   f. $\dfrac{12x^2y^3}{7xy}$

Check your answers by turning to the Appendix.



Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

In this section you evaluated powers involving rational number bases and integer exponents. The rational number bases included integers, fractions, and decimal numbers.

1. Find the puzzle "Can You Build This?" in the Appendix. Either photocopy the puzzle or pull out the page; then complete the puzzle.

Check your answers by turning to the Appendix.

When simplifying expressions that contain powers, you can apply the power law first or do what's in parentheses first (following the rules for order of operations). The result will be the same.

Example 1

Evaluate $\left(8^2\right)^3$.

Solution

$$\begin{aligned}\left(8^2\right)^3 &= 8^6 \\ &= 8\times8\times8\times8\times8\times8 \\ &= 262\,144\end{aligned} \qquad \text{or} \qquad \begin{aligned}\left(8^2\right)^3 &= 64^3 \\ &= 64\times64\times64 \\ &= 262\,144\end{aligned}$$

Example 2

Evaluate $\left(2^2\times3^3\right)^2$.

Solution

$$\begin{aligned}\left(2^2\times3^3\right)^2 &= 2^4\times3^6 \\ &= 16\times729 \\ &= 11\,664\end{aligned} \qquad \text{or} \qquad \begin{aligned}\left(2^2\times3^3\right)^2 &= (4\times27)^2 \\ &= 108^2 \\ &= 11\,664\end{aligned}$$

Example 3

Evaluate $\left(\frac{6^3}{4^2}\right)^2$.

Solution

$$\left(\frac{6^3}{4^2}\right)^2 = \frac{6^6}{4^4} = \frac{46\,656}{256} = 182.25 \qquad \text{or} \qquad \left(\frac{6^3}{4^2}\right)^2 = \left(\frac{216}{16}\right)^2 = (13.5)^2 = 182.25$$

Sometimes it's easier to simplify within the parentheses first; other times it's easier to apply the power law first.

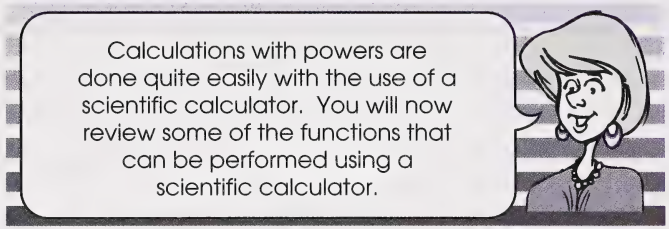
2. Evaluate each of the following using both methods of simplifying.

   a. $(5^3)^2$ b. $\left[(-2)^3\right]^2$ c. $\left[(1.5)^4\right]^0$

   d. $(4^3 \times 2^{-3})^2$ e. $\left(\frac{9^2}{3^3}\right)^2$

3. Which method in question 2 did you find easier?

Check your answers by turning to the Appendix.



The following are some functions that can be done on a calculator.

- x^y allows you to input any base with any exponent.

  For example, to find the value of 8^5, enter the following:

  8 x^y 5 =

  32768.

  Also, to evaluate 2^{-5}, enter the following:

  2 x^y 5 +/− =

  0.03125

- $\boxed{a\frac{b}{c}}$ allows you to input fractions.

For example, to find the value $\left(\frac{3}{4}\right)^2$, enter the following:

$\boxed{3}$ $\boxed{a\frac{b}{c}}$ $\boxed{4}$ $\boxed{x^y}$ $\boxed{2}$ $\boxed{=}$

0.5625

$0.5625 = \frac{9}{16}$

Also, to find the value of $\left(\frac{9}{10}\right)^{-3}$, enter the following:

$\boxed{9}$ $\boxed{a\frac{b}{c}}$ $\boxed{1}$ $\boxed{0}$ $\boxed{x^y}$ $\boxed{3}$ $\boxed{+/-}$ $\boxed{=}$

1.371742112

- $\boxed{\frac{1}{x}}$ will calculate the reciprocal (answer is in decimal form).

For example, to find the reciprocal of $\frac{4}{5}$, enter the following:

$\boxed{4}$ $\boxed{a\frac{b}{c}}$ $\boxed{5}$ $\boxed{\frac{1}{x}}$

1.25

$1.25 = \frac{5}{4}$ or $1\frac{1}{4}$

- $\boxed{\frac{1}{x}}$ can also be used to calculate the value of any number with a negative exponent.

For example, to find the value of $\left(\frac{4}{5}\right)^{-2}$, enter the following:

$\boxed{4}$ $\boxed{a\frac{b}{c}}$ $\boxed{5}$ $\boxed{x^y}$ $\boxed{2}$ $\boxed{=}$ $\boxed{\frac{1}{x}}$

1.5625

$1.5625 = \frac{25}{16}$ or $1\frac{9}{16}$

4. What does each of the following keystroke sequences represent? Do not solve.

a. $\boxed{1}$ $\boxed{1}$ $\boxed{x^y}$ $\boxed{4}$

b. $\boxed{2}$ $\boxed{a\frac{b}{c}}$ $\boxed{3}$ $\boxed{x^y}$ $\boxed{3}$ $\boxed{+/-}$

c. $\boxed{1}$ $\boxed{a\frac{b}{c}}$ $\boxed{8}$ $\boxed{x^y}$ $\boxed{4}$ $\boxed{+/-}$ $\boxed{=}$ $\boxed{x^y}$ $\boxed{2}$

5. Is there a way to input question 4.c. using fewer steps that will give the same answer?

6. Calculate all the answers to question 4.

Check your answers by turning to the Appendix.

If you have access to a computer, you may wish to view the software program *Understanding Exponents* (Neufeld and Associates) for additional help with exponents. Do Part 1: Meaning of Exponents, Part 2: Exponents in Formulas, and Part 3: The Exponent Rules; and answer as many of the Practice questions as you feel are necessary. This program may be available at your school or local library.

Enrichment

In Mathematics 8, you were shown how to calculate interest earned on money invested. This was called simple interest; but most banks or lending institutions deal in **compound interest**. This means that interest is added to the principal at the end of each calculation period (monthly, semi-annually, annually, etc.). Therefore, there is a new principal upon which interest is paid for each succeeding calculation period.

Example 1

How much would you owe on a loan of $2000 after three years if the interest rate is 8%/a compounded annually?

Solution

After one year, you would owe $160 interest ($\$2000 \times 8\%$) plus $2000 principal, or $2160.

After two years, you would owe $172.80 interest ($\$2160 \times 8\%$) plus $2160 principal, or $2332.80.

After three years, you would owe $186.62 interest ($\$2332.80 \times 8\%$) plus $2332.80 principal, or $2519.42.

1. How does this compare with simple interest?

Check your answers by turning to the Appendix.

There is a formula that you can use to calculate the amount owed with compound interest. This formula uses exponents.

$$A = P(1+r)^t$$

$$\text{amount} = \text{principal}(1 + \text{interest rate})^{(\#\text{ of compounding periods})}$$

Example 2

Use $A = P(1+r)^t$ to solve Example 1.

Solution

$$\begin{aligned} A &= P(1+r)^t \\ &= 2000(1+0.08)^3 \\ &= 2000(1.08)^3 \\ &= 2519.42 \end{aligned}$$

On a scientific calulator, the keystrokes are as follows:

2 0 0 0 × 1 • 0 8 x^y 3 =

25 19.424

With the use of a scientific calculator and the formula, it's easy to find the answer.

Use a scientific calculator to answer the following questions. Do at least question 2 and question 4.b.

2. Alfred borrowed \$750 at $7\frac{1}{2}$%/a compounded annually. How much should he pay back after two years?

3. Mr. Lau invests \$2500 on his daughter's second birthday at 6.75%/a compounded annually. How much money will he have for his daughter's university education on her eighteenth birthday?

4. **a.** If Natasha invests \$800 at 7.5%/a compounded semi-annually (every six months) for four years, explain why the calculation would look like $A = 800(1.0375)^8$ when determining her total investment.

b. Calculate the amount of Natasha's investment after four years.

c. What would Natasha's investment have been if it was compounded annually? Explain the difference.

Check your answers by turning to the Appendix.

Radioactive Decay with Spreadsheets

The worst nuclear accident in history occurred on April 26, 1986, at a nuclear power station in Chernobyl, Ukraine. One of the radioactive substances released was Iodine-131, which has a half-life of eight days. This means that one-half of a given amount of Iodine-131 will decay to another form every eight days.

Suppose 100 g of Iodine-131 were released. How long would it take for there to be less than 1 g to be remaining?

Set up a spreadsheet, using any spreadsheet program (such as *ClarisWorks*™) as follows. Double the column width for column A.

	A	B
1	Radioactive Decay	
2		
3	Amount Iodine-131	Time-Days
4		
5		
6		

You can use the following steps if you are using a *ClarisWorks*™ program on a Macintosh computer.

Step 1: Click on cell A4, and enter 100.

Step 2: Click on cell A5, and enter the formula =A4*0.5.

5. What value appears in cell A5?

Check your answers by turning to the Appendix.

Step 3: Click on cell A5; then go to the Edit menu and select Copy. Now, click on cell A6; then go to the Edit menu and select Paste.

Step 4: Repeat Step 3 for cells A7 to A12.

Step 5: Click on cell B4, and enter 0.

Step 6: Click on cell B5, and enter the formula =B4+8.

Step 7: Copy and paste the formula in Step 6 in each of cells B6 to B12.

Your spreadsheet should look as follows:

	A	B
1	Radioactive Decay	
2		
3	Amount Iodine-131 (g)	Time (days)
4	100	0
5	50	8
6	25	16
7	12.5	24
8	6.25	32
9	3.125	40
10	1.5625	48
11	0.78125	56
12	0.390625	64

6. What do you notice about the values in column A?

7. After how many days does the amount of Iodine-131 become less than 1 g?

8. Change the value in cell A4 to 1000 g. What happens to the values in cells A5 to A12?

Check your answers by turning to the Appendix.

Step 8: Click on cell C3, and enter the words Fraction Iodine-131 Remaining. (Double the column width.)

Step 9: Click on cell C4, and enter the formula =A4/1000.

Step 10: Copy and paste the formula in Step 9 in each of cells C5 to C12.

Your spreadsheet should look as follows:

	A	B	C
1	Radioactive Decay		
2			
3	Amount Iodine-131 (g)	Time (days)	Fraction Iodine-131 Remaining
4	1000	0	1
5	500	8	0.5
6	250	16	0.25
7	125	24	0.125
8	62.5	32	0.0625
9	31.25	40	0.03125
10	15.625	48	0.015625
11	7.8125	56	0.0078125
12	3.90625	64	0.00390625

9. What is the relationship between the values in each of cells C4 to C12 and its respective cell in column A?

Check your answers by turning to the Appendix.

You may wish to try various initial amounts of Iodine-131 in cell A4. Be sure to make the appropriate adjustment in the formula in cells C4 to C12.

10. Following are some different kinds of problems dealing with exponents. Some will be quite challenging, and others will not be as difficult. Try to answer **at least three** out of the seven questions.

a. Determine two values of n such that $n^2 = 2^n$.

b. Which of the following does
$6^6 + 6^6 + 6^6 + 6^6 + 6^6 + 6^6$ equal?

A. 6^6
B. 6^7
C. 36^6
D. 6^{36}
E. 36^{36}

c. If $(-1-n)^3 = 1$, then n equals what number?

d. Write $4^3 \times 8^2 \times 16^3$ as a single power; then calculate.

e. 9999^4 is between which two powers of 10?

f. Find the sum of the digits in the value of $100^{25} - 25$.

g. What are the last three digits if you evaluate 5^{101}?

Check your answers by turning to the Appendix.

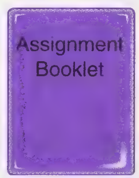
Conclusion

In this section you worked with exponential notation. You increased your knowledge of powers by examining patterns that helped develop rules for calculating powers. You then used these rules to calculate the solutions to questions and problems involving powers.

The modern colour computer monitor can produce 256 tones of each of the three basic colours. In total, 16 777 216 colours can be produced. Your new knowledge of powers allowed you to express this total in exponential form in more than one way and show how this total is calculated.

In the next section you will apply some of your knowledge of powers as you look at a different way of expressing some other very large and very small numbers.

Assignment



You are now ready to complete the assignment for Section 1.

Section 2: Scientific Notation

NASA

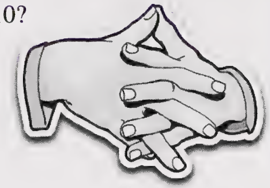
Saturn As It Appears Through the Hubble Space Telescope

Saturn's rings make it one of the most intriguing bodies in the sky. Saturn orbits the Sun at an average distance of 1 427 000 000 km; it has ten satellites (moons)—the most distant moon being 13 000 000 km from Saturn; and the diameter of its entire ring system is about 270 000 km.

How do these measurements compare with Earth's and those of other planets? If you write them in scientific notation, you can make these comparisons quite easily.

In this section, you will use your knowledge of powers to help you express these large numbers (along with some very small numbers) in scientific notation. You will also perform calculations with numbers in scientific notation and compare their relative measures.

Activity 1: Powers of Ten

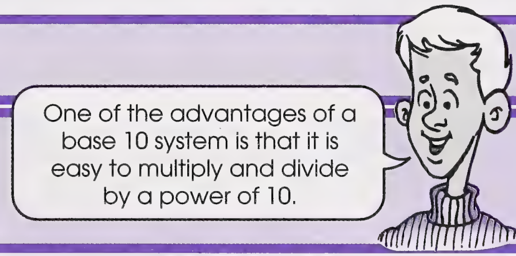


Have you ever thought about the number 10? You have 10 fingers and 10 toes. Perhaps that is how our base 10 number system was developed. An understanding of powers of ten is very important for writing numbers in scientific notation.

The distance to Sirrus, one of the brightest stars in the northern hemisphere, is about 85 000 000 000 000 km. Using scientific notation, this is written as 8.5×10^{13} km. In this activity you will express numbers both as powers of 10 and in standard form.

The following chart shows several powers of ten.

Name	Standard Form	Power Form
one million	1 000 000	10^6
one hundred thousand	100 000	10^5
ten thousand	10 000	10^4
one thousand	1000	10^3
one hundred	100	10^2
ten	10	10^1
one	1	10^0
one tenth	$\frac{1}{10}$ or 0.1	10^{-1}
one hundredth	$\frac{1}{100}$ or 0.01	10^{-2}
one thousandth	$\frac{1}{1000}$ or 0.001	10^{-3}
one ten thousandth	$\frac{1}{10\,000}$ or 0.0001	10^{-4}
one hundred thousandth	$\frac{1}{100\,000}$ or 0.000 01	10^{-5}
one millionth	$\frac{1}{1\,000\,000}$ or 0.000 001	10^{-6}



1. Use $100 \div 1000$ to answer the following:

a. Write $100 \div 1000$ as a fraction.

b. Write the fraction from question 1.a. in lowest terms.

c. Use a scientific calculator to convert the fraction from question 1.b. to a decimal.

d. Use the power form in the previous chart to convert 100 and 1000 to power form.

e. Apply the quotient rule to divide the powers.

f. Is the answer to question 1.e. equivalent to the answers to questions 1.b. and 1.c.?

Check your answers by turning to the Appendix.

Now look at a similar example.

Example

What is $10 \div 1000$?

Solution

$$10 \div 1000 = \frac{10}{1000} = \frac{1}{100} \text{ or } 0.01$$

or

$$10 \div 1000 = 10^1 \div 10^3 = 10^{1-3} = 10^{-2}$$

It is clear that $10^{-2} = \frac{1}{100}$ or 0.01.

You can confirm this by using a pattern.

Standard Form	Power Form	Pattern
10 000	10^4	$\div 10$
1000	10^3	$\div 10$
100	10^2	$\div 10$
10	10^1	$\div 10$
1	10^0	$\div 10$
$\frac{1}{10}$ or 0.1	10^{-1}	$\div 10$
$\frac{1}{100}$ or 0.01	10^{-2}	

Notice that the exponents 2 and -2 are opposites and that the values of 10^2 and 10^{-2} are reciprocals.

$$10^2 \times 10^{-2} = 10^{2+(-2)} = 10^0 = 1$$

or

$$10^2 \times 10^{-2} = 100 \times \frac{1}{100} = 1$$

2. How many zeros are there between the decimal and the first non-zero digit in the decimal form of 10^{-10}?

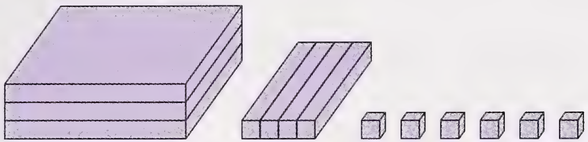
3. Find the puzzle "Daffynition Decoder" in the Appendix. Either photocopy the puzzle or pull out the page; then complete the puzzle.

Check your answers by turning to the Appendix.

Expanded Form

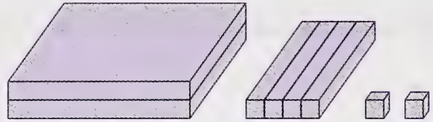
In previous mathematics courses, you have modelled numbers using base 10 blocks. You have discovered that numbers can be written in **expanded form**.

If  represents 1, then 346 can be modelled as follows:



$$\underset{\text{standard form}}{346} = \underbrace{3 \times 100 + 4 \times 10 + 6 \times 1}_{\text{expanded form}}$$

If  represents 1, then 2.42 can be modelled as follows:

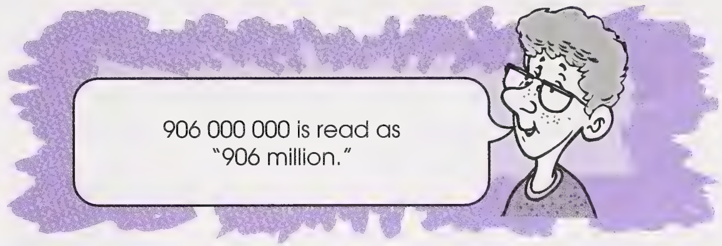


$$\underset{\text{standard form}}{2.42} = \underbrace{2 \times 1 + 4 \times 0.1 + 2 \times 0.01}_{\text{expanded form}}$$

In this activity you will write the expanded form of a number in a shorter form using powers of ten.

Example 1

How do you write 906 000 000 in expanded form?

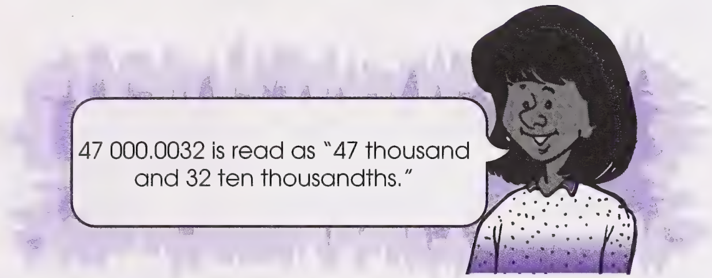


Solution

$$\begin{aligned} 906\,000\,000 &= 9 \times 100\,000\,000 + 6 \times 1\,000\,000 \\ &= \left(9 \times 10^8\right) + \left(6 \times 10^6\right) \end{aligned}$$

Example 2

What is 47 000.0032 in expanded form?



Solution

$$47\,000.0032=\left(4\times10^{4}\right)+\left(7\times10^{3}\right)+\left(3\times10^{-3}\right)+\left(2\times10^{-4}\right)$$

Example 3

Write the standard form of the following:

$$\left(8\times10^{5}\right)+\left(6\times10^{4}\right)+\left(3\times10^{0}\right)+\left(7\times10^{-2}\right)+\left(4\times10^{-5}\right)$$

Solution

$$\left(8\times10^{5}\right)+\left(6\times10^{4}\right)+\left(3\times10^{0}\right)+\left(7\times10^{-2}\right)+\left(4\times10^{-5}\right)$$
$$=860\,003.070\,04$$

4. Write each of the following in expanded form using powers of ten.

a. 97 billion
b. 0.000 000 089
c. three million four hundred five and six thousandths
d. 708 000 000.009
e. 1 trillion

5. Write the name for each of the following:

a. 6×10^{8}
b. 0.000 006
c. $\left(8\times10^{5}\right)+\left(6\times10^{3}\right)+\left(1\times10^{0}\right)+\left(4\times10^{-3}\right)$
$+\left(2\times10^{-4}\right)$
d. 10^{-9}

6. Write the standard form of the following:

a. 54 thousandths
b. $\left(8\times10^{4}\right)+\left(3\times10^{-2}\right)$
c. 10^{9}
d. $\left(6\times10^{5}\right)+\left(4\times10^{4}\right)+\left(8\times10^{-3}\right)$

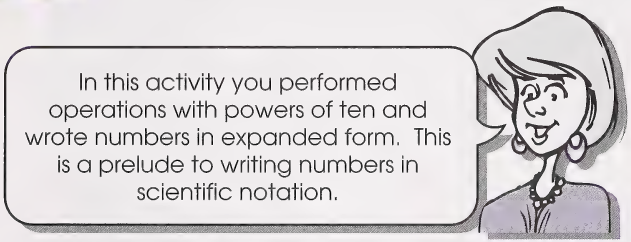
7. Copy and complete the following charts in your notebook.

Expanded Form	Standard Form
8.6×10^1	
8.6×10^2	
8.6×10^3	
8.6×10^4	

Expanded Form	Standard Form
8.6×10^{-1}	
8.6×10^{-2}	
8.6×10^{-3}	
8.6×10^{-4}	

8. The diameter of a virus is 0.000 000 1 m. Write this number as a power of ten.

Check your answers by turning to the Appendix.



Activity 2: The Basics of Scientific Notation

The distance between Saturn and the sun is 1 427 000 000 km. (This is not an exact distance. The first three digits are accurate; the fourth digit is an estimate; and the zeros are place holders.) The mass of an oxygen atom is 0.000 000 000 000 000 000 000 024 9 g.

Very large and very small numbers (like the distance from Saturn to the Sun and the mass of an oxygen atom) are awkward to write and difficult to read or compare. But, there is an alternative. These numbers can be written in **scientific notation**. Do you recall from previous mathematics courses how to write numbers in scientific notation? Do you recall the definition of scientific notation?

Scientific notation is a notation for writing very large or very small numbers as a product of a number (between 1 and 10) and a power of 10.

The following table will give you a review of changing standard form to scientific notation.

1. Copy and complete the following table in your notebook. The first two and the last one have been completed for you.

Standard Form	Product Form	Scientific Notation
8 640 000	$8.64 \times 1\,000\,000$	8.64×10^{6}
864 000	$8.64 \times 100\,000$	8.64×10^{5}
86 400	$8.64 \times$	
8640	$8.64 \times$	
864	$8.64 \times$	
86.4	$8.64 \times$	
8.64	$8.64 \times$	
0.864	$8.64 \times$	
0.0864	$8.64 \times$	
0.008 64	$8.64 \times$	
0.000 864	8.64×0.0001	8.64×10^{-4}

2. Normally, a number like 8640 is not written in scientific notation. Why wouldn't you write 8640 in scientific notation?

3. Change the distance between Saturn and the sun and the mass of an oxygen atom, given previously, to scientific notation.

Check your answers by turning to the Appendix.

The following steps show an easier way to convert very large numbers to scientific notation.

Step 1: Starting from the right side of the first non-zero digit, count the number of digits to the end of the number. The total count is the exponent of the power of 10. Since the count is to the right, the exponent is positive.

Step 2: Write the number in scientific notation form as a product of the number between 1 and 10 and the power of ten.

Example 1

Write 13 050 000 in scientific notation.

Solution

13 050 000

Since the count is 7 to the right, the exponent of the power of 10 is 7.

$$\therefore\ 13\,050\,000 = 1.305 \times 10^{7}$$

The following steps show an easier way to convert very small numbers to scientific notation.

Step 1: Starting from the right of the first non-zero digit, count the number of digits to the decimal. The total count is the exponent of the power of 10. Since the count is to the left, the exponent is negative.

Step 2: Write the number in scientific notation form as a product of the number between 1 and 10 and the power of 10.

Example 2

Write 0.000 000 000 014 7 in scientific notation.

Solution

0.000 000 000 01 47

Since the count is 11 to the left, the exponent of the power of 10 is −11.

$\therefore\ 0.000\,000\,000\,014\,7 = 1.47 \times 10^{-11}$

4. Copy each statement in your notebook and complete it so that the number is written in scientific notation.

   a. $24\,000\,000 = 2.4 \times 10^{\square}$
   b. $0.000\,000\,43 = 4.3 \times 10^{\square}$
   c. $54\,600\,000\,000 = \square \times 10^{10}$
   d. $0.000\,000\,000\,039 = \square \times 10^{-11}$
   e. $147\,000\,000 = \square \times 10^{\square}$
   f. $0.000\,000\,083 = \square \times 10^{\square}$

5. Which of the following numbers are not in scientific notation? Explain. Rewrite them so that they are in scientific notation.

   a. 4.8×2^{10}
   b. 52×10^{-9}
   c. 7×10
   d. 1×10^{-12}
   e. 0.45×10^{15}

6. Write each of the following in scientific notation.

   a. 48 000 000 000
   b. 0.000 000 007 2
   c. 1 000 000 000 000 000

7. Write these numbers in standard form.

   a. 7.12×10^{8}
   b. 4.2×10^{-10}
   c. 1×10^{5}

You may find a scientific calculator very useful when doing multiplication and division with numbers in scientific notation. Use a scientific calculator to answer questions 8 and 9.

8. Enter $187\,000 \times 567\,000$ on a scientific calculator. What result do you get?

9. Enter $0.0006 \div 30\,000$ on a scientific calculator. What result do you get?

Check your answers by turning to the Appendix.

Scientific calculators express very large and very small numbers in scientific notation. However, these calculators may not display the multiplication sign or the base of the power of 10. For example,

5.138^{08}

means 5.138×10^8.

10. Write the following calculator displays in scientific notation.

a. 4.35^{06}

b. 3.27^{-05}

11. Why is the answer to question 8 displayed in scientific notation?

12. What is the largest number your scientific calculator can display in standard form?

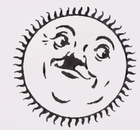
The answer to question 9 written in standard form is 0.000 000 02. Although this number contains less than the maximum of ten digits, it is written in scientific notation. To discover why, answer questions 13 and 14.

13. Perform the following calculations involving decimals on your calculator, and write out the result in the display.

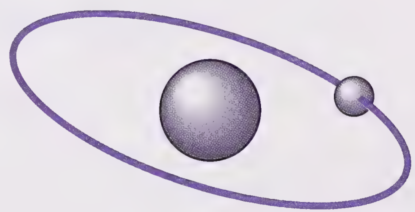
a. $0.03 \div 1$ **b.** $0.03 \div 10$ **c.** $0.03 \div 100$
d. 0.03×1 **e.** 0.03×0.1 **f.** 0.03×0.01

14. What pattern do you see in the results?

15. The temperature at the centre of the Sun reaches about 13 000 000°C. Write this temperature in scientific notation.

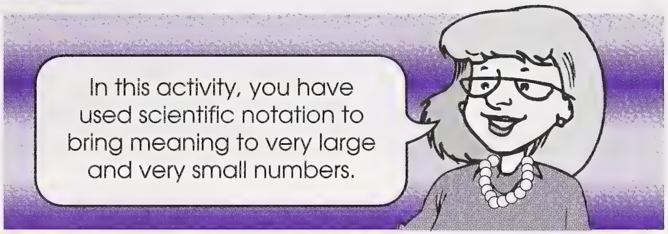


16. The mass of a hydrogen atom is about 0.000 000 000 000 000 000 000 001 67 g. Write this mass in scientific notation.

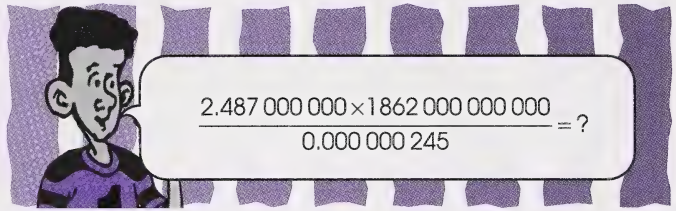


17. The great pyramid (Horizon of Khufu), with an original height of 146.7 m and a base line of 230 m, covers about 5 ha (or 13 acres). It has been estimated that its construction required a permanent work force of 4000 people over 30 years to manoeuvre the 2 300 000 blocks into position. Calculate the total number of person-hours of work. Write this number in scientific notation to two decimal places.

Check your answers by turning to the Appendix.



Activity 3: Computations with Numbers in Scientific Notation



Would you be concerned if you had to do this calculation using paper and pencil? If you express numbers like these in scientific notation, then the calculation becomes easier. In this activity you will discover how to execute calculations like these by writing the numbers in scientific notation and then performing the indicated operations or by using a scientific calculator.

Multiplying

Scientific notation can be used in multiplying very large and very small numbers. It makes it easier to do calculations involving numbers that occur with distances in outer space or with numbers involving atomic masses.

The mass of Earth is about 6.0×10^{24} kg. It has been determined that the mass of the Sun is about 3.3×10^{5} times greater than the mass of Earth. Calculate the mass of the Sun.


NASA

To calculate the mass of the Sun, you need to be able to multiply numbers in scientifc notation. Set up the multiplication as follows:

$$\left(6.0 \times 10^{24}\right) \times \left(3.3 \times 10^{5}\right)$$

Rearrange the numbers so you have a product of the decimal numbers and of the powers.

$$(6.0 \times 3.3) \times \left(10^{24} \times 10^{5}\right)$$

Use a calculator to multiply the decimal numbers. (**Remember:** Add the exponents when multiplying powers.)

$$\begin{aligned}(6.0 \times 3.3) \times \left(10^{24} \times 10^{5}\right) &= 19.8 \times 10^{24+5} \\ &= 19.8 \times 10^{29} \\ &= 1.98 \times 10^{1} \times 10^{29} \\ &= 1.98 \times 10^{1+29} \\ &= 1.98 \times 10^{30}\end{aligned}$$

Therefore, the mass of the Sun is about 1.98×10^{30} kg.

Remember: When a number is written in scientific notation, it is written as the product of a number (equal to or greater than 1 and less than 10) and a power of 10.

You can use a scientifc calculator to multiply numbers in scientific notation directly. To use a scientific calculator directly, you need to be able to enter the numbers in scientific notation.

To enter a number in scientific notation, use EXP between the decimal number and the power. Do not use × between the decimal number and the power. **Note:** Some calculators have a key labelled EE rather than EXP.

The following shows how to determine the mass of the Sun using a scientific calculator.

6 EXP 2 4 ×

6.24

3 • 3 EXP 5

3.3 05

=

1.98 30

Therefore, the mass of the Sun is approximately 1.98×10^{30} kg.

Note: If you are using a scientific calculator to solve problems involving scientific notation, then always estimate your answer first.

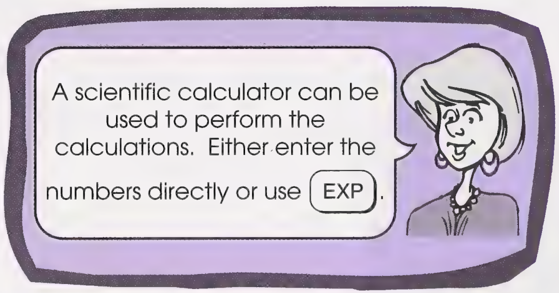
You can use scientific notation to help approximate an answer.

Example 1

Use scientific notation to help approximate the answer to $64\,182 \times 389\,475$; then calculate the answer using a scientific calculator.

Solution

$$\begin{aligned}64\,182 \times 389\,475 &\doteq 60\,000 \times 400\,000\\ &\doteq (6 \times 10^4) \times (4 \times 10^5)\\ &\doteq (6 \times 4) \times (10^4 \times 10^5)\\ &\doteq 24 \times 10^{4+5}\\ &\doteq 24 \times 10^9\\ &\doteq 24\,000\,000\,000\end{aligned}$$



6 4 1 8 2 ×

64 182.

3 8 9 4 7 5

389475.

=

2.499728445 10

or

6 • 4 1 8 2 EXP 4 ×

6.4 182 04

3 • 8 9 4 7 5 EXP 5

3.89475 05

=

2.499728445 10

The answer to $64\,182 \times 389\,475$ is about 2.50×10^{10}.

Example 2

AOSTRA

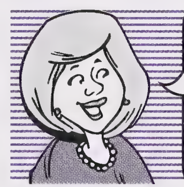
Chemists estimate that 6.02×10^{23} molecules of hydrogen gas occupy 24.8 L at 25°C at normal atmospheric pressure. If one molecule of hydrogen gas has a mass of 3.34×10^{-24} g, what is the mass of 24.8 L of hydrogen gas?

Solution

$$\left(6.02 \times 10^{23}\right) \times \left(3.34 \times 10^{-24}\right) = (6.02 \times 3.34) \times \left(10^{23} \times 10^{-24}\right)$$
$$= 20.1068 \times 10^{23+(-24)}$$
$$= 20.1068 \times 10^{-1}$$
$$= 2.010\,68 \times 10^{1} \times 10^{-1}$$
$$= 2.010\,68 \times 10^{1+(-1)}$$
$$= 2.010\,68 \times 10^{0}$$
$$= 2.010\,68$$

This is not in scientific notation because 20.1068 is not between 1 and 10.

The mass of 24.8 L of hydrogen gas is 2.010 68 g.



Again, a scientific calculator can be used to help you with the calculation. Be sure to estimate the answer before using a calculator, since it is easy to press a wrong key.

Estimate.

$$(6.02\times10^{23})\times(3.34\times10^{-24}) \doteq (6\times10^{23})\times(3\times10^{-24})$$
$$\doteq (6\times3)\times(10^{23}\times10^{-24})$$
$$\doteq 18\times10^{23+(-24)}$$
$$\doteq 18\times10^{-1}$$
$$\doteq 1.8\times10^{1}\times10^{-1}$$
$$\doteq 1.8\times10^{0}$$
$$\doteq 1.8$$

The mass of 24.8 L of hydrogen is approximately 1.8 g.

6 • 0 2 EXP 2 3 ×

6.02 23

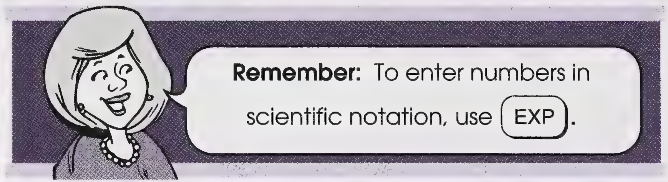
3 • 3 4 EXP 2 4 +/−

3.34 -24

=

2.01068

You may use a scientific calculator to help you with the calculations in questions 1 to 4.



1. Multiply the following. Write the answer in scientific notation to two decimal places.

   a. $(8.14\times10^{-4})\times(9.25\times10^{6})$

   b. $(7.73\times10^{5})\times(2.68\times10^{-1})$

   c. $(3.91\times10^{11})\times(1.84\times10^{-19})$

2. Write out the keystroke sequence you would use on a scientific calculator to perform the following operations.

   a. $(1.2\times10^{-6})\times(6.4\times10^{11})$

   b. $(9.7\times10^{21})\times(3.7\times10^{-14})$

3. Write out the following calculator displays in scientific notation.

a. 7.94 12

b. -3.22 06

c. 9.01 -15

4. The speed of light is about 300 000 000 m/s. How far does light travel in 1 h?

Check your answers by turning to the Appendix.

Dividing

Distances in space are so great that various units other than kilometres are used. Astronomers use light years and astronomical units (AU) to measure distances in space. An astronomical unit is the mean distance from Earth to the Sun, a distance of 1.496×10^8 km.

To determine the distance in AU for the other planets in the solar system, you need the mean distance from each planet to the Sun and to be able to divide numbers in scientific notation. The following chart gives the mean distance from each of the nine planets in the solar system to the Sun.

Planet	Mean Distance to Sun (km)	Mean Distance to Sun (AU)
Mercury	5.7900×10^7	
Venus	1.0820×10^8	
Earth	1.4960×10^8	1.000
Mars	2.2790×10^8	
Jupiter	7.7840×10^8	
Saturn	1.4238×10^9	
Uranus	2.8687×10^9	
Neptune	4.4921×10^9	
Pluto	5.9265×10^9	

For instance, to find the mean distance from Mars to the Sun in AU, divide the mean distance of Mars (in kilometres) by the mean distance of Earth.

$$\frac{2.2790\times10^{8}}{1.4960\times10^{8}}=\frac{2.2790}{1.4960}\times10^{8-8}$$
$$\doteq 1.523\times10^{0}$$
$$\doteq 1.523 \text{ AU}$$

Using a scientific calculator, enter the keystrokes as follows:

2 • 2 7 9 EXP 8

÷ 1 • 4 9 6 EXP 8 =

1.523395722

The distance from Mars to the Sun is 1.523 AU.

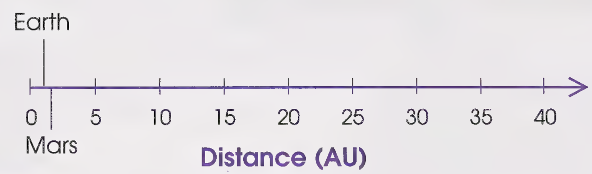
5. Copy and complete the previous chart. Round your answers to three decimal places. (**Hint:** You can save time by entering the distance from Earth to the Sun into the calculator's memory and dividing each mean distance to the Sun using memory recall.)

6. Use the completed chart from question 5 to answer the following questions.

a. Which planet is about halfway between the Sun and Uranus?

b. Approximately how many times further from the Sun is Pluto than Earth?

7. Plot the planets on a number line like the following. Earth and Mars have been done for you.



Check your answers by turning to the Appendix.

When dividing numbers in scientific notation, divide the decimal numbers and divide the powers of 10. (**Remember:** Subtract the exponents when dividing powers.) Make sure your answer is written in proper scientific notation. Study the following examples and answer the questions that follow.

Example

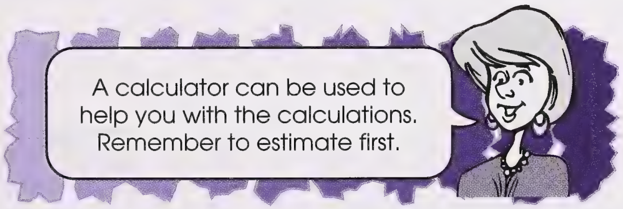
If 1 200 000 silkworm eggs have a mass of 1000 g, what is the mass of one silkworm egg?

Solution

$$\begin{aligned}\frac{1000}{1\,200\,000} &= \frac{1\times10^{3}}{1.2\times10^{6}}\\ &= (1\div1.2)\times\left(10^{3-6}\right)\\ &= 0.8\bar{3}\times10^{-3}\\ &= 8.\bar{3}\times10^{-1}\times10^{-3}\\ &= 8.\bar{3}\times10^{-1+(-3)}\\ &= 8.\bar{3}\times10^{-4}\end{aligned}$$

This is not scientific notation because the number is not between 1 and 10.

The mass of one silkworm egg is $8.\bar{3}\times10^{-4}$ g.



Estimate.

$$\begin{aligned}\frac{1000}{1\,200\,000} &= \frac{1\times10^{3}}{1\times10^{6}}\\ &= \frac{1}{1}\times10^{3-6}\\ &= 1\times10^{-3}\end{aligned}$$

The mass of one silkworm egg is approximately 1×10^{-3} g.

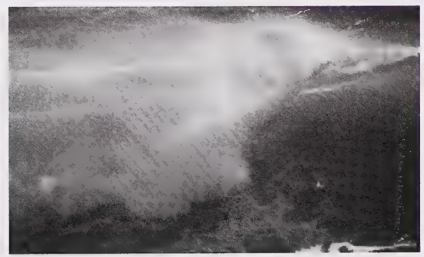
Using a scientific calculator, enter the keystrokes as follows:

1 EXP 3 ÷ 1 • 2 EXP 6 =

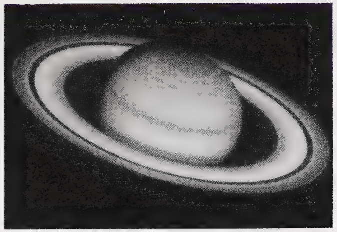
8.333333333^{-04}

Use a scientific calculator to answer questions 8 to 13.

8. Earth's oceans contain many dissolved salts and minerals. There is about 1 kg of gold in 0.25 km^3 of sea water. If the total volume of the oceans is about $1.4\times10^{9}\text{ km}^3$, how much gold is there in the oceans?



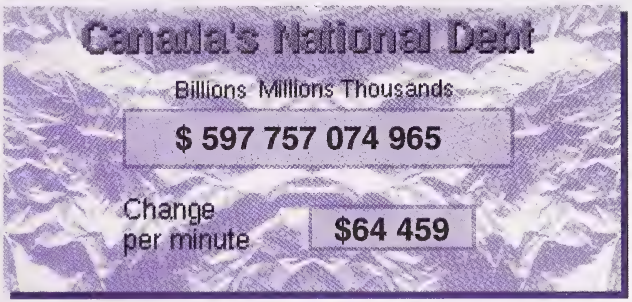
9. Light travels at a speed of 1.08×10^9 km/h. If Saturn is 1.427×10^9 km from the Sun, then how long will it take for light from the Sun to reach Saturn?



NASA

10. A molecule of water has a mass of about 3×10^{-26} kg. How many molecules of water are there in 1 L of water? (Remember, $1 \text{ L} = 1000 \text{ cm}^3 = 1000 \text{ g}$.)

11. On October 25, 1996 Canada's national debt was \$597 757 074 965 and increasing at a rate of \$64 459 per minute. The population of Canada was 30 326 902 at the time.



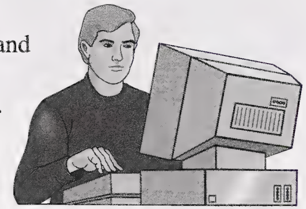
a. Approximately how much is each person's share of the national debt?

b. Calculate the amount the debt increases after one day and after one month (30 d) based on the rate of increase at that time.

c. Calculate the amount of the debt at the present time using the given rate of change. Compare your calculated amount to the actual federal debt. You can find the actual federal debt through the Internet or find the most recently printed amount through a library.

d. How has Canada fared in reducing its national debt?

12. Have somebody time you as you multiply 486 by 325 using paper and pencil. Some computers do this same calculation in 0.000 000 1 s.



a. How many times faster is the computer?

b. How much time (in years) would you need to do what the computer can do in 1 s?

13. State each answer in scientific notation. Where required, write your answer correct to two decimal places.

a. $\dfrac{8\,500\,000 \times 74\,000\,000}{280\,000\,000\,000}$

b. $\dfrac{\left(4 \times 10^6\right) \times \left(5 \times 10^8\right) \times \left(3 \times 10^7\right)}{6 \times 10^{10}}$

c. $\dfrac{0.000\ 000\ 4 \times 85\ 000\ 000}{0.000\ 000\ 17}$

d. $\dfrac{40 \text{ billion} \times 35 \text{ millionths}}{25 \text{ thousandths}}$

Check your answers by turning to the Appendix.

Now Try This

Solve the following challenge problem.

14. The number of particles in the universe was estimated by the physicist, Sir Arthur Eddington, to be 33×2^{259}. Write this number in scientific notation to two decimal places.

Check your answers by turning to the Appendix.

If you have access to the Internet, answer question 15. Use the Internet to find information on the human world population. Use the Internet's search engine to find the world's population and a world population counter.

15. **a.** Write the human world population in scientific notation.

    b. Use the 30 s update on the world population counter to find the increase in the human population in 1 min.

    c. Based on the rate of change of population in question 15.b., what will the change in human population be after one year and after ten years? Write your answers in scientific notation.

    d. How long will it take the human population to double? Round your answer to the nearest year.

Check your answers by turning to the Appendix.

Did You Know?

Mary Fairfax Somerville

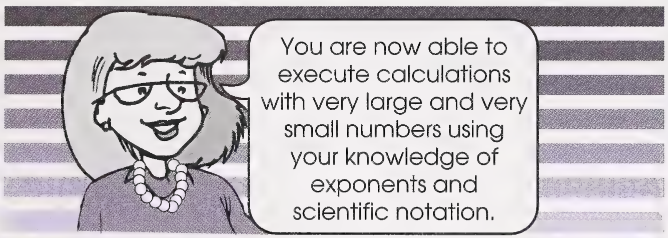
Mary Fairfax Somerville almost lost out on learning mathematics because she was a girl. She was born in Scotland in 1780 during a time when girls there were supposed to learn only such things as cooking and sewing. She did cook and sew, but she was much happier when she could run and play on the beach near her home.

Her parents tried to make her more of a lady by sending her to a girls' school. There she had to wear a tight corset, learn how to paint, and memorize a whole page in a dictionary. She hated this school.

Once, in a magazine, she saw an arithmetic problem. She got so excited about it that she wanted to learn mathematics—something girls didn't do! She happened to hear someone talking about Euclid, one of the great Greek mathematicians, and she decided to try to buy a copy of his book. Even this was hard for her to do because girls did not go into stores. Her brother's teacher got the book for her. She studied it very hard, and a whole new world of ideas opened up to her. One day she helped her brother with a problem, much to the surprise of his teacher.

After this, Mary studied many other things on her own. By the time she married her cousin, William Somerville, in 1812, she had learned about calculus and astronomy. Calculus is a hard part of mathematics, and astronomy is the study of the way the stars move. Dr. Somerville introduced his wife to many scientists and mathematicians who were happy to talk to Mary about her studies.

She wrote several books about mathematics and science. She also found time to raise a family. Mary Fairfax Somerville was still studying mathematics on the day she died at the age of ninety-two. [1]

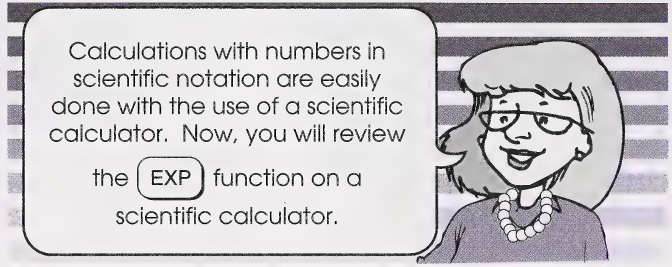


[1] Merle Mitchell, *Mathematical History: Activities, Puzzles, Stories, and Games* (Reston, VA: National Council of Mathematics Teachers, 1978), 13. Reprinted by permission.

Follow-up Activities

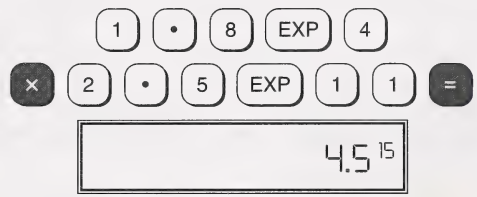
If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

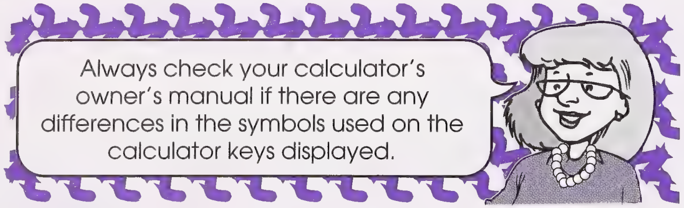
Extra Help



- EXP allows you to input any number in scientific notation.

For example, to evaluate $(1.8 \times 10^{4}) \times (2.5 \times 10^{11})$, enter the following:





1. What does the following sequence of keystrokes represent?

   9 EXP 4 ÷ 3 EXP 6 +/–

2. What will the calculator display show after each of these inputs? **Do not** use your calculator to help you.

   a. 5 EXP 6 × 4 EXP 3 =

   b. 1 • 5 EXP 7 × 2 EXP 4 =

   c. 8 • 4 EXP 3 ÷
   2 • 1 EXP 9 +/– =

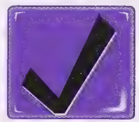
3. The following puzzles will provide an interesting way to review scientific notation. Do at least two of the puzzles. You may do all four.

   a. Find the puzzle "What Has Long Hair and Purple Feet?" in the Appendix. Either photocopy the puzzle or pull out the page; then complete the puzzle.

   b. Find the puzzle "Find a Match" in the Appendix. Either photocopy the puzzle or pull out the page; then complete the puzzle.

   c. Find the puzzle "What Did Dr. Watson Say About Sherlock?" in the Appendix. Either photocopy the puzzle or pull out the page; then complete the puzzle.

   d. Find the puzzle "Did You Hear About…" in the Appendix. Either photocopy the puzzle or pull out the page; then complete the puzzle.



Check your answers by turning to the Appendix.

If you have access to a computer, you may wish to view the software program *Understanding Exponents* (Neufeld and Associates) for additional help with exponents. Do Part 4: Scientific Notation, and answer as many of the Practice questions as you feel are necessary. This program may be available at your school or local library.

Enrichment

You are familiar with the terms "thousand," "million," and "billion."

- The standard form of one thousand is 1000. The power form of one thousand is 10^3.
- The standard form of one million is 1 000 000. The power form of one million is 10^6.
- The standard form of one billion is 1 000 000 000. The power form of one billion is 10^9.

1. Research and provide the names for each of the following powers of 10.

   a. 10^9 **b.** 10^{12} **c.** 10^{15}
   d. 10^{18} **e.** 10^{21} **f.** 10^{24}
   g. 10^{27} **h.** 10^{30} **i.** 10^{33}

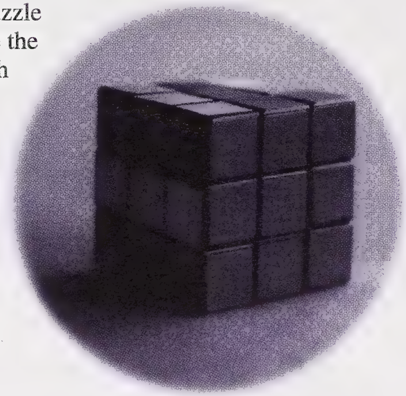
2. Research and provide the powers of 10 of the following names.

   a. undecillion **b.** duodecillion
   c. tredecillion **d.** quattuordecillion
   e. quindecillion **f.** sexdecillion
   g. septendecillion **h.** octodecillion
   i. novemdecillion

3. Write the powers for each of the following

   a. ten quadrillion
   b. one hundred decillion
   c. ten billion

4. Erno Rubik invented a puzzle that requires you to rotate the pieces of a cube until each face of the cube is one colour. Amazingly, some people can solve the puzzle in seconds. This cube can be rearranged in over 43 quintillion different ways. Write this number in scientific notation.



Check your answers by turning to the Appendix.

Now Try This

You may wish to use the Internet to search for examples that use scientific notation. Type the words "scientific notation" in an Internet search engine and you will get a number of articles on this topic. Check out a number of sites for more information on scientific notation.

Conclusion

NASA

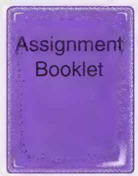
Earth's Moon

In this section, you have discovered how to express very large and very small numbers in scientific notation. Scientific notation is a very compact way of writing numbers that you often see used in science (such as measures of very large distances in the universe and measures of very minute things that you can only see with the aid of powerful microscopes).

Earth has one moon, it is 3.84×10^5 km away from Earth. Saturn has ten moons; the farthest moon, Phoebe, is 1.3×10^7 km from Saturn. It is $\left(1.3 \times 10^7\right) \div \left(3.84 \times 10^5\right) \doteq 33.8$ or almost 34 times farther away from Saturn than Earth's moon is from Earth.

Using scientific notation, along with a scientific calculator, enables you to perform calculations involving very large and very small numbers easily.

Assignment



You are now ready to complete the assignment for Section 2.

Module Summary


NASA

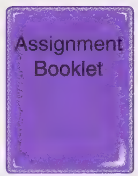
In this module, you have expanded your knowledge of powers. You are now able to multiply and divide with powers and simplify a variety of expressions with powers in different forms. You used your knowledge of powers to write numbers in scientific notation and to multiply and divide numbers written in scientific notation. Your calculator should be an even more powerful tool now that you have become familiar with more of its functions.

You can use your new knowledge of exponential and scientific notation to help you understand all the technological advances involving large and small numbers.

Satellites travel beyond the fringes of the solar system. Space telescopes enable us to see even further. Electron microscopes and computer imaging enable us to see into the atom.

The universe as you know it is becoming both exponentially larger and exponentially smaller.

Final Module Assignment



You are now ready to complete the final module assignment.

APPENDIX

Glossary

Suggested Answers

Puzzles

Glossary

Base (of a power): the factor being multiplied by itself

$3^4 \leftarrow$ base

Compound interest: interest earned or charged on an amount of money and added to the principal to earn or charge more interest in the following year

Expanded form: a form of a number written as the sum of the product of each digit and its place value

$$326 = (3\times100)+(2\times10)+(6\times1)$$

$$326 = \left(3\times10^2\right)+\left(2\times10^1\right)+\left(6\times10^0\right)$$

Exponent (of a power): the number of times the base occurs as a factor

$3^{4 \leftarrow \text{exponent}}$

Exponential form: a form of a number written as a power

The exponential form of 81 is 3^4.

Factored form: a form of a number written as a product of its factors

$$3^4 = 3\times3\times3\times3$$

Power: a product of equal factors

3^4 is the fourth power of 3. 3^4 means $3\times3\times3\times3$.

Scientific notation: a notation for writing very large and very small numbers as a product of a number (between 1 and 10) and a power of 10

$$32\,000\,000\,000\,000 = 3.2\times10^{13}$$

$$0.000\,000\,58 = 5.8\times10^{-7}$$

Sextillion: a number written with 1 followed by 21 zeros $\left(10^{21}\right)$

Standard form: the usual form of a number

7856 is in standard form.

Variable: a letter used to represent or take the place of a number in a mathematical expression

$$2\underset{\substack{\uparrow\\ \text{variable}}}{a}+7$$

Suggested Answers

Section 1: Activity 1

1.

Number of Folds	Number of Layers
0	1
1	2
2	4
3	8
4	16

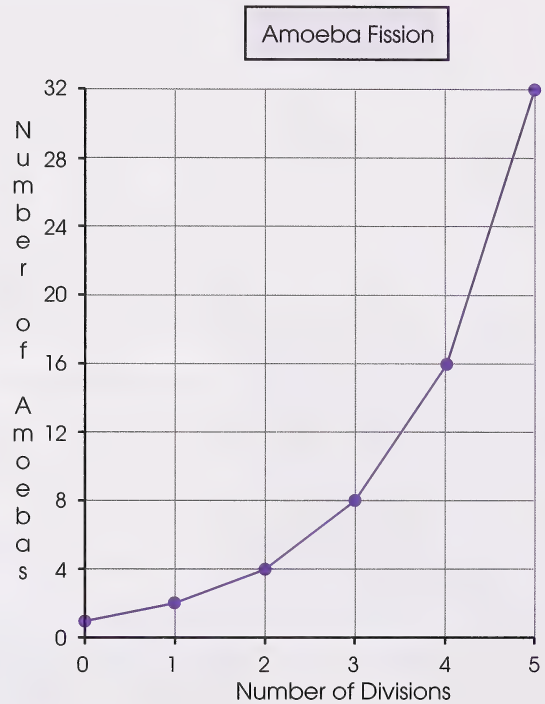
2. a. 1 amoeba b. 2 amoebas c. 4 amoebas

   d. 8 amoebas e. 16 amoebas f. 32 amoebas

3. Your chart should look similar to the following.

	A	B
1	Number of Divisions	Number of Amoebas
2	0	1
3	1	2
4	2	4
5	3	8
6	4	16
7	5	32

4. Your graph should look similar to the following.



5. a.

Number of Divisions	Number of Amoebas at Each Division		
	Standard Form	Factored Form	Power Form
1	2	2	2^1
2	4	2×2	2^2
3	8	$2 \times 2 \times 2$	2^3
4	16	$2 \times 2 \times 2 \times 2$	2^4
5	32	$2 \times 2 \times 2 \times 2 \times 2$	2^5

b. There will be 64 amoebas.

c. There will be 2^6 amoebas.

d. The factored form of 2^6 is $2 \times 2 \times 2 \times 2 \times 2 \times 2$.

e. The number of factors in the factored form is equal to the exponent in the power form.

6. a. $16 = 2 \times 2 \times 2 \times 2$
$= 2^4$

b. $8 = 2 \times 2 \times 2$
$= 2^3$

c. $32 = 2 \times 2 \times 2 \times 2 \times 2$
$= 2^5$

7. a. $9 = 3 \times 3$
$= 3^2$

b. $81 = 3 \times 3 \times 3 \times 3$
$= 3^4$

c. $27 = 3 \times 3 \times 3$
$= 3^3$

8. a. $49 = 7 \times 7$
$= 7^2$

b. $144 = 12 \times 12$
$= 12^2$

c. $27 = 3 \times 3 \times 3$
$= 3^3$

d. $10\,000 = 10 \times 10 \times 10 \times 10$
$= 10^4$

9. Answers will vary. Some examples are given.

a. seven cubed, seven exponent three, or seven to the third power

b. negative six to the fourth power

c. one point five to the fifth power

10. a. There is 1 amoeba after zero divisions.

b. The power used to describe the number of amoebas after zero divisions is 2^0. If you follow the pattern of exponents in the power form in reverse, the next number for the exponent should be a 0.

11. 0 x^y 0 =

When you enter 0^0 into a calculator, the result is an error message.

12. **a.** $2^0 = 1$

b. $(-12)^0 = 1$

c. $\left(\frac{1}{2}\right)^0 = 1$

d. $3^0 \times 4 = 1 \times 4$
$= 4$

13.

Power Form	Standard Form	Pattern
2^2	4	
		Divide by 2.
2^1	2	
		Divide by 2.
2^0	1	
		Divide by 2.
2^{-1}	$\frac{1}{2}$	
		Divide by 2.
2^{-2}	$\frac{1}{4}$	
		Divide by 2.
2^{-3}	$\frac{1}{8}$	
		Divide by 2.
2^{-4}	$\frac{1}{16}$	
		Divide by 2.
2^{-5}	$\frac{1}{32}$	
		Divide by 2.
2^{-6}	$\frac{1}{64}$	

14. **a.** $2^2 = 4$ and $2^{-2} = \frac{1}{2^2}$ or $\frac{1}{4}$

They are reciprocals of each other.

b. $2^1 = 2$ and $2^{-1} = \frac{1}{2^1}$ or $\frac{1}{2}$

They are reciprocals of each other.

15. The exponents are opposites. 2 is the opposite of -2, and 1 is the opposite of -1.

16. **a.** $4^{-3} = \frac{1}{4^3}$

b. $7^{-4} = \frac{1}{7^4}$

c. $(0.6)^{-5} = \frac{1}{(0.6)^5}$

17. **a.** $\frac{1}{8} \times \frac{1}{8} \times \frac{1}{8} = \frac{1}{8^3}$
$= 8^{-3}$

b. $\frac{1}{(-3)(-3)(-3)(-3)} = \frac{1}{(-3)^4}$
$= (-3)^{-4}$

18. **a.** $5^6 = 5 \times 5 \times 5 \times 5 \times 5 \times 5$
$= 15\,625$

b. $13^3 = 13 \times 13 \times 13$
$= 2197$

c. $39^4 = 39 \times 39 \times 39 \times 39$
$= 2\,313\,441$

d. $1.7^3 = 1.7 \times 1.7 \times 1.7$
$= 4.913$

19. **a.** Using the Automatic Constant

6 × ×

6.

=

36.

=

216.

=

1296.

=

7776.

$\therefore\ 6^5 = 7776$

Using the Power Key

6 x^y 5 =

7776.

$\therefore\ 6^5 = 7776$

b. Using the Automatic Constant

7 × ×

7.

=

49.

=

343.

=

2401.

$\therefore\ 7^4 = 2401$

Using the Power Key

7 x^y 4 =

2401.

$\therefore\ 7^4 = 2401$

c. **Using the Automatic Constant**

1 • 2 × ×

1.2

=

1.44

=

1.728

$\therefore\ 1.2^3 = 1.728$

Using the Power Key

1 • 2 x^y 3 =

1.728

$\therefore\ 1.2^3 = 1.728$

20. Answers will vary. You may prefer the power key since the calculator does the steps in fewer keystrokes.

21. **a.**
$$\left(\frac{2}{3}\right)^5 = \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3} = \frac{4}{9}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3} = \frac{8}{27}\times\frac{2}{3}\times\frac{2}{3} = \frac{16}{81}\times\frac{2}{3} = \frac{32}{243}$$

or

$$\left(\frac{2}{3}\right)^5 = \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3} = \frac{4}{9}\times\frac{4}{9}\times\frac{2}{3} = \frac{16}{81}\times\frac{2}{3} = \frac{32}{243}$$

b.
$$\left(\frac{4}{7}\right)^3 = \frac{4}{7}\times\frac{4}{7}\times\frac{4}{7} = \frac{16}{49}\times\frac{4}{7} = \frac{64}{343}$$

c.
$$\left(3\frac{1}{3}\right)^2 = \left(\frac{10}{3}\right)^2 = \frac{10}{3}\times\frac{10}{3} = \frac{100}{9} \text{ or } 11\frac{1}{9}$$

22. **a.** **Using the Automatic Constant**

1 $a\frac{b}{c}$ 3 × ×

1 ⌋ 3.

=

1 ⌋ 9.

=

1 ⌋ 27.

=

1 ⌋ 81.

Therefore, $\left(\frac{1}{3}\right)^4 = \frac{1}{81}$ or 0.012, rounded to three decimal places.

Using the Power Key

1 $a\frac{b}{c}$ 3 x^y 4 =

0.012345679

Therefore, $\left(\frac{1}{3}\right)^4 = 0.012$, rounded to three decimal places.

b. **Using the Automatic Constant**

3 $a\frac{b}{c}$ 4 × ×

3 ⌋ 4.

=

9 ⌋ 16.

=

27 ⌋ 64.

=

81 ⌋ 256.

=

243 ⌋ 1024.

Therefore, $\left(\frac{3}{4}\right)^5 = \frac{243}{1024}$ or 0.237, rounded to three decimal places.

Using the Power Key

3 $a\frac{b}{c}$ 4 x^y 5 =

0.237304687

Therefore, $\left(\frac{3}{4}\right)^5 = 0.237$, rounded to three decimal places.

c. ### Using the Automatic Constant

2 $a\frac{b}{c}$ 3 $a\frac{b}{c}$ 8 × ×

2 ⌟ 3 ⌟ 8.

=

5 ⌟ 41 ⌟ 64.

=

13 ⌟ 203 ⌟ 512.

Therefore, $\left(2\frac{3}{8}\right)^3 = 13\frac{203}{512}$ or 13.396, rounded to three decimal places.

Using the Power Key

2 $a\frac{b}{c}$ 3 $a\frac{b}{c}$ 8 x^y 3 =

13.39648438

Therefore, $\left(2\frac{3}{8}\right)^3 = 13.396$, rounded to three decimal places.

23. **a.** Answers may vary. Most calculator users prefer using the power key since the calculator performs all the steps for any exponent.

b. When you use the automatic constant, you have to keep track of how many times you have used the equal key.

c. Using the power key is more efficient.

24. **a.** $3^2 = 9$ and $2^3 = 8$

Thus, 3^2 is greater.

b. $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$ and $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$

Thus, $\left(\frac{1}{2}\right)^3$ is greater.

25. Look at the exponent. If the exponent is even, then the answer is positive. If the exponent is odd, then the answer is negative.

26. **a.** $(-6)^3 = (-6)\times(-6)\times(-6)$
$= -216$

b. $(-0.5)^2 = (-0.5)\times(-0.5)$
$= 0.25$

c. $(-3)^5 = (-3)\times(-3)\times(-3)\times(-3)\times(-3)$
$= -243$

27. [+/–] changes the sign of the number entered.

28. **a.** [4] [+/–] [x^y] [5] [=]

-1024.

Therefore, $(-4)^5 = -1024$.

b. [•] [8] [5] [+/–] [x^y] [3] [=]

-0.614125

Therefore, $(-0.85)^3 = -0.614$, rounded to three decimal places.

c. [1] [•] [7] [+/–] [x^y] [4] [=]

8.3521

Therefore, $(-1.7)^4 = 8.352$, rounded to three decimal places.

29. $(-5)^4 =$ [(] [5] [+/–] [)] [x^y] [4] [=]

625.

$-5^4 =$ [5] [+/–] [x^y] [4] [=]

625.

30. Yes, the calculator answers are the same.

31. **a.** The calculator is taking the -5 as one entry and applying the exponent to -5 as if it were in brackets.

b. To get the calculator to give the same answer, you would need to use the [+/–] after you obtain the answer, 625. The keystrokes are as follows.

[5] [+/–] [x^y] [4] [=]

625.

[+/–]

-625.

32. **a.** negative **b.** negative **c.** positive

d. negative **e.** positive

33. **a.** $$\begin{aligned} -8^2 &= -(8\times 8) \\ &= -64 \end{aligned}$$

b. $$\begin{aligned} (-8)^3 &= (-8)\times(-8)\times(-8) \\ &= -512 \end{aligned}$$

c. $$\begin{aligned} (-8)^4 &= (-8)\times(-8)\times(-8)\times(-8) \\ &= 4096 \end{aligned}$$

d. $$\begin{aligned} -(-3)^2 &= -[(-3)\times(-3)] \\ &= -(+9) \\ &= -9 \end{aligned}$$

e. $$\begin{aligned} -(-2)^5 &= -[(-2)\times(-2)\times(-2)\times(-2)\times(-2)] \\ &= -(-32) \\ &= 32 \end{aligned}$$

34. **a.** $$\begin{aligned} (-4)^2 &= (-4)\times(-4) \\ &= 16 \end{aligned}$$

$$\begin{aligned} -4^2 &= -(4\times 4) \\ &= -16 \end{aligned}$$

Therefore, $(-4)^2$ is greater.

b. $$\begin{aligned} -3^4 &= -(3\times 3\times 3\times 3) \\ &= -81 \end{aligned}$$

$$\begin{aligned} -(-3)^4 &= -[(-3)\times(-3)\times(-3)\times(-3)] \\ &= -(81) \\ &= -81 \end{aligned}$$

They are equal.

35. **a.** $$\begin{aligned} 10^{-3} &= \left(\frac{1}{10}\right)^3 \\ &= \frac{1}{10}\times\frac{1}{10}\times\frac{1}{10} \\ &= \frac{1}{1000} \text{ or } 0.001 \end{aligned}$$

b. $$\begin{aligned} 2^{-5} &= \left(\frac{1}{2}\right)^5 \\ &= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} \\ &= \frac{1}{32} \text{ or } 0.031\,25 \end{aligned}$$

c. $$\begin{aligned} \left(\frac{1}{4}\right)^{-2} &= 4^2 \\ &= 4\times 4 \\ &= 16 \end{aligned}$$

d. $$\begin{aligned} 5^{-2} &= \left(\frac{1}{5}\right)^2 \\ &= \frac{1}{5}\times\frac{1}{5} \\ &= \frac{1}{25} \text{ or } 0.04 \end{aligned}$$

e. $(-3)^{-3} = \left(-\frac{1}{3}\right)^3$

$= \left(-\frac{1}{3}\right)\times\left(-\frac{1}{3}\right)\times\left(-\frac{1}{3}\right)$

$= -\frac{1}{27}$ or $-0.\overline{037}$

f. $(0.5)^{-4} = \left(\frac{1}{0.5}\right)^4$

$= \frac{1}{0.5}\times\frac{1}{0.5}\times\frac{1}{0.5}\times\frac{1}{0.5}$

$= \frac{1}{0.0625}$

$= 16$

or $(0.5)^{-4} = \left(\frac{1}{2}\right)^{-4}$

$= 2^4$

$= 2\times2\times2\times2$

$= 16$

36. a. $4^{-2} = \frac{1}{4^2}$

$= \frac{1}{16}$

$5^{-2} = \frac{1}{5^2}$

$= \frac{1}{25}$

$\therefore\ 4^{-2} > 5^{-2}$

b. $4^{-10} = \left(\frac{1}{4}\right)^{10}$

$(-4)^{10} = 4^{10}$ since 10 is even

$\therefore\ 4^{-10} < (-4)^{10}$

c. $8^{-10} = \left(\frac{1}{8}\right)^{10}$

$\therefore\ 8^{-10} > 0$

d. $-11^0 = -1 \qquad 11^0 = 1$

$\therefore\ -11^0 < 11^0$

e. $3^{-5} = \left(\frac{1}{3}\right)^5$

$\therefore\ 3^{-5} = \left(\frac{1}{3}\right)^5$

f. $4^{-9} = \left(\frac{1}{4}\right)^9 \qquad (-4)^9 = -4^9$

$\therefore\ 4^{-9} > (-4)^9$

37. A power with a negative exponent is the reciprocal of a power with a positive exponent.

38. a. $4^{-1}\times2^{-1} = \frac{1}{4}\times\frac{1}{2}$

$= \frac{1}{8}$

b. $4^{-2}+4^0 = \left(\frac{1}{4}\right)^2 + 1$

$= \frac{1}{16}+1$

$= 1\frac{1}{16}$

c. $\left(\frac{1}{8}\right)^{-2} + \left(\frac{1}{10}\right)^{-2} = 8^2 + 10^2$

$= 64 + 100$

$= 164$

d. $\left(\frac{4}{5}\right)^{-2}+\left(\frac{2}{3}\right)^{-4}=\left(\frac{5}{4}\right)^{2}+\left(\frac{3}{2}\right)^{4}$

$=\frac{25}{16}+\frac{81}{16}$

$=\frac{106}{16}$ or 6.625

39. a.

$$\begin{array}{r|l} 3 & 81 \\ 3 & 27 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array}$$

There are four divisions; thus, $81=3^4$.

b.

$$\begin{array}{r|l} 2 & 32 \\ 2 & 16 \\ 2 & 8 \\ 2 & 4 \\ 2 & 2 \\ & 1 \end{array}$$

There are five divisions; thus, $32=2^5$.

c.

$$\begin{array}{r|l} 11 & 1331 \\ 11 & 121 \\ 11 & 11 \\ & 1 \end{array}$$

There are three divisions; thus, $1331=11^3$.

40. a. $256=2^8$ **b.** $3^8=6561$

c. $(1.4)^4=3.8416$ **d.** $-78\,125=(-5)^7$

41. a.

$3^1=3$ $3^5=243$

$3^2=9$ $3^6=729$

$3^3=27$ $3^7=2187$

$3^4=81$ $3^8=6561$

The pattern is
3, 9, 7, 1, 3, 9, 7, 1,… .

b.

$4^1=4$

$4^2=16$

$4^3=64$

$4^4=256$

The pattern is
4, 6, 4, 6,… .

c.

$5^1=5$

$5^2=25$

$5^3=125$

$5^4=625$

The pattern is
5, 5, 5, 5,… .

d.

$7^1=7$ $7^5=16\,807$

$7^2=49$ $7^6=117\,649$

$7^3=343$ $7^7=823\,543$

$7^4=2401$ $7^8=5\,764\,801$

The pattern is
7, 9, 3, 1, 7, 9, 3, 1,… .

42. a. Since the base is 2, the cycles are the same as in the example. Since the exponent is 25, there are six complete cycles and one step into the seventh cycle. The first step ends in 2. Therefore, the last digit will be 2.

b. From the answer to question 40.a., the pattern is $3, 9, 7, 1, 3, 9, 7, 1, \ldots$. Again, the cycles repeat in groups of four. Since the exponent is 18, there are four complete cycles and two steps into the fifth cycle. The second step ends in 9. Therefore, the last digit will be 9.

c. From the answer to question 40.d., the cycles repeat in groups of four. Since the exponent is 11, there are two complete cycles and three steps into the third cycle. The third step ends in 3. Therefore, the last digit will be 3.

d. From the answer to question 40.c., the last digit always ends with 5. Therefore, the last digit will be 5.

43. 2^5 ends in 2; 3^5 ends in 3; 4^5 ends in 4; and so on. According to the pattern, all standard forms end in the number that is the base of the power when the exponent is 5.

44. The last digit in the standard form will be the same as the last digit in the base.

a. The last digit in 13^5 is 3.

b. The last digit in 32^5 is 2.

c. The last digit in 17^5 is 7.

45. a. $1^2, 2^2, 3^2, 4^2, 5^2$

b. The next three numbers will be 36, 49, and 64.

Now Try This

46. The number of loonies on a square on the board is given by a power of 2.

$2^0 = 1$ ← first square

$2^1 = 2$ ← second square

$2^2 = 4$ ← third square

$2^3 = 8$ ← fourth square

$\vdots$

$2^{19} = 524\ 288$ ← twentieth square

$2^{20} = 1\ 048\ 576$ ← twenty-first square

Therefore, the first square to have over $1 000 000 on it is the twenty-first square.

47. The smallest number for which twice the number is smaller than the number squared is 3. This can be shown using the following pattern.

$1 \times 2 = 2$ and $1^2 = 1$

$2 \times 2 = 4$ and $2^2 = 4$

$3 \times 2 = 6$ and $3^2 = 9$

$4 \times 2 = 8$ and $4^2 = 16$

Section 1: Activity 2

1.

Expression	Factored Form	Power Form
$3^4 \times 3^3$	$(3\times3\times3\times3)\times(3\times3\times3)$ $=3\times3\times3\times3\times3\times3\times3$	3^7
$4^2 \times 4^3$	$(4\times4)\times(4\times4\times4)$ $=4\times4\times4\times4\times4$	4^5
5×5^3	$5\times(5\times5\times5)$ $=5\times5\times5\times5$	5^4
$2^2\times2^3\times2^4$	$(2\times2)\times(2\times2\times2)$ $\times(2\times2\times2\times2)$ $=2\times2\times2\times2\times2\times2\times2\times2\times2$	2^9

2. You can obtain the answers in question 1 by adding the exponents. For example, $4^2 \times 4^3 = 4^{2+3} = 4^5$.

3. The shortcut does not work for $3^2 \times 4^5$ because the bases are different. You can only add the exponents when multiplying powers with identical bases.

4. **a.** $2^3 \times 2^7 = 2^{3+7}$
$= 2^{10}$

b. $(-5)^8 \times (-5)^{10} = (-5)^{8+10}$
$= (-5)^{18}$

c. $\left(\frac{1}{2}\right)^5 \times \left(\frac{1}{2}\right)^6 = \left(\frac{1}{2}\right)^{5+6}$
$= \left(\frac{1}{2}\right)^{11}$

d. This cannot be written as a single power because the bases are not the same.

e. $8^3 \times 8^4 \times 8^5 = 8^{3+4+5}$
$= 8^{12}$

f. $3^2 \times 3^9 \times 5 \times 5^4 = 3^{2+9} \times 5^{1+4}$
$= 3^{11} \times 5^5$

5. **a.** $8^2 \times 8^{-3} = 8^{2+(-3)}$
$= 8^{-1}$
$= \frac{1}{8}$

b. $2^{-3} \times 2^{-2} = 2^{-3+(-2)}$
$= 2^{-5}$
$= \left(\frac{1}{2}\right)^5$
$= \frac{1}{32}$

c. $\left(\frac{1}{4}\right)\times\left(\frac{1}{4}\right)^{-3}=\left(\frac{1}{4}\right)^{1+(-3)}$

$=\left(\frac{1}{4}\right)^{-2}$

$=4^{2}$

$=16$

6. **a.** $10^{15}\times10^{4}=10^{15+4}$

$=10^{19}$

The electron will make 10^{19} orbits.

b. $10^{15}\times10^{11}=10^{15+11}$

$=10^{26}$

The electron will make 10^{26} orbits.

7. $10^{5}\times10^{4}=10^{5+4}$

$=10^{9}$

The signal is boosted by a factor of 10^{9} after it has passed through both amplifiers.

8. No, you could not spend either fortune in a lifetime.

9.

Expression	Factored Form	Power Form
$5^6 \div 5^2$	$(5\times5\times5\times5\times5\times5)\div(5\times5)$ $=\dfrac{\overset{1}{\cancel{5}}\times\overset{1}{\cancel{5}}\times5\times5\times5\times5}{\underset{1}{\cancel{5}}\times\underset{1}{\cancel{5}}}$ $=5\times5\times5\times5$	5^4
$2^5 \div 2^3$	$(2\times2\times2\times2\times2)\div(2\times2\times2)$ $=\dfrac{\overset{1}{\cancel{2}}\times\overset{1}{\cancel{2}}\times\overset{1}{\cancel{2}}\times2\times2}{\underset{1}{\cancel{2}}\times\underset{1}{\cancel{2}}\times\underset{1}{\cancel{2}}}$ $=2\times2$	2^2
$4^6 \div 4^5$	$(4\times4\times4\times4\times4\times4)$ $\div(4\times4\times4\times4\times4)$ $=\dfrac{\overset{1}{\cancel{4}}\times\overset{1}{\cancel{4}}\times\overset{1}{\cancel{4}}\times\overset{1}{\cancel{4}}\times\overset{1}{\cancel{4}}\times4}{\underset{1}{\cancel{4}}\times\underset{1}{\cancel{4}}\times\underset{1}{\cancel{4}}\times\underset{1}{\cancel{4}}\times\underset{1}{\cancel{4}}}$ $=4$	4^1
$3^7 \div 3^4$	$(3\times3\times3\times3\times3\times3\times3)$ $\div(3\times3\times3\times3)$ $=\dfrac{\overset{1}{\cancel{3}}\times\overset{1}{\cancel{3}}\times\overset{1}{\cancel{3}}\times\overset{1}{\cancel{3}}\times3\times3\times3}{\underset{1}{\cancel{3}}\times\underset{1}{\cancel{3}}\times\underset{1}{\cancel{3}}\times\underset{1}{\cancel{3}}}$ $=3\times3\times3$	3^3

10. You can obtain the answers in question 9 by subtracting the exponents. For example, $2^5 \div 2^3 = 2^{5-3} = 2^2$.

11. The shortcut only works if the bases of the powers being divided are the same.

12. **a.** $3^6 \div 3^2 = 3^{6-2}$
$= 3^4$

b. $(-5)^7 \div (-5)^3 = (-5)^{7-3}$
$= (-5)^4$

c. $(0.8)^9 \div (0.8)^8 = (0.8)^{9-8}$
$= (0.8)^1$ or 0.8

d. $\frac{5^7}{5^7} = 5^{7-7}$
$= 5^0$ or 1

e. This cannot be simplified because the bases are not the same.

f. $\left(\frac{1}{2}\right)^7 \div \left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^{7-2}$
$= \left(\frac{1}{2}\right)^5$

13. **a.** $5^2 \div 5^4 = 5^{2-4}$
$= 5^{-2}$

b. $(-3)^2 \div (-3)^5 = (-3)^{2-5}$
$= (-3)^{-3}$

c. $4^{-2} \div 4^{-4} = 4^{-2-(-4)}$
$= 4^{-2+4}$
$= 4^2$

Now Try This

14. **a.** $\frac{8^2 \times 8^3}{8^4} = 8^{2+3-4}$
$= 8^{5-4}$
$= 8^1$ or 8

b. $\frac{9^2 \times 9^3 \times 9^4}{9 \times 9^5} = \frac{9^{2+3+4}}{9^{1+5}}$
$= \frac{9^9}{9^6}$
$= 9^{9-6}$
$= 9^3$

c. $\left(4^2 \times 4^6 \times 4\right) \div \left(4^5 \times 4 \times 4\right) = \left(4^{2+6+1}\right) \div \left(4^{5+1+1}\right)$
$= 4^9 \div 4^7$
$= 4^{9-7}$
$= 4^2$

15. $10^{27} \div 10^{22} = 10^{27-22}$
$= 10^5$
$= 100\,000$

The mass of the Sun is about 10^5 or 100 000 times greater than the mass of Earth.

16. **a.** p^7 **b.** m^6 **c.** n^6 **d.** p^5 **e.** a^3

Section 1: Activity 3

1.

Expression	Factored Form	Power Form
$(5^3)^2$	$5^3 \times 5^3$ $=(5\times5\times5)\times(5\times5\times5)$ $=5\times5\times5\times5\times5\times5$	5^6
$(7^2)^3$	$7^2\times7^2\times7^2$ $=(7\times7)\times(7\times7)\times(7\times7)$ $=7\times7\times7\times7\times7\times7$	7^6
$(3^4)^2$	$3^4\times3^4$ $=(3\times3\times3\times3)$ $\times(3\times3\times3\times3)$ $=3\times3\times3\times3\times3\times3\times3\times3$	3^8
$(2^3)^4$	$2^3\times2^3\times2^3\times2^3$ $=(2\times2\times2)\times(2\times2\times2)$ $\times(2\times2\times2)\times(2\times2\times2)$ $=2\times2\times2\times2\times2\times2$ $\times2\times2\times2\times2\times2\times2$	2^{12}

2. You can obtain the answers in question 1 by multiplying the exponents. For example, $\left(7^2\right)^3 = 7^{2\times3} = 7^6$.

3. **a.** $\left(3^5\right)^4 = 3^{5\times4}$
$= 3^{20}$

b. $\left[(-2)^3\right]^7 = (-2)^{3\times7}$
$= (-2)^{21}$

c. $\left[(0.2)^5\right]^3 = (0.2)^{5\times3}$
$= (0.2)^{15}$

d. $\left[\left(\frac{5}{8}\right)^2\right]^6 = \left(\frac{5}{8}\right)^{2\times6}$
$= \left(\frac{5}{8}\right)^{12}$

e. $\left(4^2\right)^{-1} = 4^{2\times(-1)}$
$= 4^{-2}$
$= \left(\frac{1}{4}\right)^2$

f. $\left(10^{-2}\right)^{-3} = 10^{(-2)\times(-3)}$
$= 10^6$

4. **a.** $\left(10^{10}\right)^{10} = 10^{10\times10}$
$= 10^{100}$

b. The exponent for a googplex would be 1 followed by 100 zeros.

5. No, a scientific calculator cannot evaluate 10^{100}. You get an error message displayed. The largest number you can enter in most scientific calculators is 10^{99}.

6. $(3^4)^2 = (3\times3\times3\times3)^2$

$= (3\times3\times3\times3)\times(3\times3\times3\times3)$ ← There are eight factors of 3.

$(3^2)^4 = (3\times3)^4$

$= (3\times3)\times(3\times3)\times(3\times3)\times(3\times3)$ ← There are eight factors of 3.

Since $(3^4)^2$ and $(3^2)^4$ have the same number of factors of 3, they are equivalent.

7. **a.** $4^9 = (2^2)^9$

$= 2^{2\times9}$

$= 2^{18}$

$\therefore\ 4^9 = 2^{18}$

b. $16^2 = (2^4)^2$

$= 2^{4\times2}$

$= 2^8$

$\therefore\ 16^2 = 2^8$

c. $3^{12} = 3^{3\times4}$

$= (3^3)^4$

$= 27^4$

$\therefore\ 3^{12} = 27^4$

8. **a.** $4^9 = (2^2)^9$

$= 2^{18}$

Thus, 2^{20} is greater.

b. $27^2 = (3^3)^2$

$= 3^6$

Thus, 3^7 is greater.

c. $16^5 = (2^4)^5$

$= 2^{20}$

They are equal.

9. $2^{55} = (2^5)^{11}$

$= 32^{11}$

$3^{44} = (3^4)^{11}$

$= 81^{11}$

$4^{33} = (4^3)^{11}$

$= 64^{11}$

$5^{22} = (5^2)^{11}$

$= 25^{11}$

Therefore, the arrangement, from smallest to largest, is 5^{22}, 2^{55}, 4^{33}, and 3^{44}.

10.

Expression	Factored Form	Power Form
$(6^2\times4^3)^3$	$(6^2\times4^3)\times(6^2\times4^3)\times(6^2\times4^3)$ $=6^2\times6^2\times6^2\times4^3\times4^3\times4^3$	$6^6\times4^9$
$(5^4\times2^7)^2$	$(5^4\times2^7)\times(5^4\times2^7)$ $=5^4\times5^4\times2^7\times2^7$	$5^8\times2^{14}$
$(8\times7^4)^3$	$(8\times7^4)\times(8\times7^4)\times(8\times7^4)$ $=8\times8\times8\times7^4\times7^4\times7^4$	$8^3\times7^{12}$
$(3^5\times4^6)^2$	$(3^5\times4^6)\times(3^5\times4^6)$ $=3^5\times3^5\times4^6\times4^6$	$3^{10}\times4^{12}$

11. You can obtain the same answers in question 10 by multiplying each of the exponents inside the brackets by the exponent outside the brackets. For example,

$$\left(5^4 \times 2^7\right)^2 = 5^{4\times2} \times 2^{7\times2} = 5^8 \times 2^{14}.$$

12. **a.** $\left(8^7 \times 6^2\right)^4 = 8^{7\times4} \times 6^{2\times4}$

$= 8^{28} \times 6^8$

b. $\left[(-3)^2 \times (-4)^7\right]^3 = (-3)^{2\times3} \times (-4)^{7\times3}$

$= (-3)^6 \times (-4)^{21}$

c. $\left(7 \times 4^2\right)^5 = 7^{1\times5} \times 4^{2\times5}$

$= 7^5 \times 4^{10}$

d. $\left[(0.5)^4 \times (1.4)^3\right]^2 = (0.5)^{4\times2} \times (1.4)^{3\times2}$

$= (0.5)^8 \times (1.4)^6$

e. $\left(5^{-6} \times 3^7\right)^{-2} = 5^{(-6)\times(-2)} \times 3^{7\times(-2)}$

$= 5^{12} \times 3^{-14}$

$= 5^{12} \times \left(\frac{1}{3}\right)^{14}$

f. $\left[(-3)^2 \times \left(\frac{1}{3}\right)^2\right]^{-3} = (-3)^{2\times(-3)} \times \left(\frac{1}{3}\right)^{2\times(-3)}$

$= (-3)^{-6} \times \left(\frac{1}{3}\right)^{-6}$

$= \left(-\frac{1}{3}\right)^6 \times 3^6$

13. **a.** 6 x^y 2 × 3 x^y 4 = x^y 2 =

8503056.

The value of $\left(6^2 \times 3^4\right)^2$ is 8 503 056.

b. • 3 x^y 2 × 2 • 6 x^y 3 = x^y 2 +/– =

0.399645468

The value of $\left[(0.3)^2 \times (2.6)^3\right]^{-2}$ is 0.40, rounded to two decimal places.

14. Evaluate question 12.d.

[•] [5] [x^y] [4] [×] [1] [•] [4] [x^y] [3]
[=] [x^y] [2] [=]

0.0294 1225

The value of $\left[(0.5)^4 \times (1.4)^3\right]^2$ is 0.029 412 25.

Evaluate the answer to question 12.d.

[•] [5] [x^y] [8]
[×] [1] [•] [4] [x^y] [6] [=]

0.0294 1225

The value of $(0.5)^8 \times (1.4)^6$ is 0.029 412 25.

Yes, the answers are the same.

15.

Expression	Factored Form	Power Form
$\left(\frac{8^2}{4^3}\right)^3$	$\left(\frac{8^2}{4^3}\right)\times\left(\frac{8^2}{4^3}\right)\times\left(\frac{8^2}{4^3}\right)$ $=\frac{8^2\times8^2\times8^2}{4^3\times4^3\times4^3}$	$\frac{8^6}{4^9}$
$\left(\frac{5^4}{3^2}\right)^2$	$\left(\frac{5^4}{3^2}\right)\times\left(\frac{5^4}{3^2}\right)$ $=\frac{5^4\times5^4}{3^2\times3^2}$	$\frac{5^8}{3^4}$
$\left(\frac{4^7}{2^5}\right)^3$	$\left(\frac{4^7}{2^5}\right)\times\left(\frac{4^7}{2^5}\right)\times\left(\frac{4^7}{2^5}\right)$ $=\frac{4^7\times4^7\times4^7}{2^5\times2^5\times2^5}$	$\frac{4^{21}}{2^{15}}$

16. You can obtain the answers in question 15 by multiplying each of the exponents inside the brackets by the exponent outside the brackets. For example, $\left(\frac{5^4}{3^2}\right)^2 = \frac{5^{4\times2}}{3^{2\times2}} = \frac{5^8}{3^4}$.

17. **a.** $\left(\frac{8^4}{2^3}\right)^3 = \frac{8^{4\times3}}{2^{3\times3}}$
$= \frac{8^{12}}{2^9}$

b. $\left(\frac{7^4}{3^5}\right)^6 = \frac{7^{4\times6}}{3^{5\times6}}$
$= \frac{7^{24}}{3^{30}}$

c. $$\left(\frac{4^7}{3^4}\right)^{-3} = \frac{4^{7\times(-3)}}{3^{4\times(-3)}} = \frac{4^{-21}}{3^{-12}} = \left(\frac{1}{4}\right)^{21} \times 3^{12} = \frac{3^{12}}{4^{21}}$$

d. $$\left[\frac{(-5)^{-2}}{(-5)^{-6}}\right]^{-2} = \frac{(-5)^{(-2)\times(-2)}}{(-5)^{(-6)\times(-2)}} = \frac{(-5)^4}{(-5)^{12}} = (-5)^{4-12} = (-5)^{-8}$$

18. Evaluate question 17.b.

7 x^y 4 ÷ 3 x^y 5 = x^y 6 =

930497.7316

The value of question 17.b. is 930 497.7316.

Evaluate the answer to question 17.b.

7 x^y 2 4 ÷ 3 x^y 3 0 =

930497.7316

The value of the answer to question 17.b. is 930 497.7316.

Yes, the answers are the same. The expressions are equivalent.

19. **a.** $(a^2)^3 = a^{2\times3} = a^6$

b. $(y^3)^4 = y^{3\times4} = y^{12}$

c. $(m^4 \times n^3)^2 = m^{4\times2} \times n^{3\times2} = m^8 \times n^6$

d. $(a^2b^2)^3 = a^{2\times3}b^{2\times3} = a^6b^6$

e. $\left(\frac{c^2}{b^3}\right)^2 = \frac{c^{2\times2}}{b^{3\times2}}$

$= \frac{c^4}{b^6}$

f. $\left(a^4 \div b^2\right)^3 = a^{4\times3} \div b^{2\times3}$

$= a^{12} \div b^6$

20. a. $n^8 \div n^4 = 81$

$n^{8-4} = 81$

$n^4 = 81$

$n^4 = 3^4$

Therefore, $n = 3$.

b. $\left(n^3\right)^2 = 729$

$n^{3\times2} = 729$

$n^6 = 729$

$n^6 = 3^6$

Therefore, $n = 3$.

c. $n^3 \times n^2 = 32$

$n^{3+2} = 32$

$n^5 = 32$

$n^5 = 2^5$

Therefore, $n = 2$.

d. $\frac{n^5 \times n^2}{n^4} = 125$

$n^{5+2-4} = 125$

$n^3 = 125$

$n^3 = 5^3$

Therefore, $n = 5$.

21. a. Try $n = 2$.

LS	RS
$2^4 \times 2^5$	2^9
$= 16 \times 32$	$= 512$
$= 512$	

LS = RS

Therefore, 2 is a correct value for *n*.

b. Try $n = 5$.

LS	RS
$5^4 \times 5^5$	5^9
$= 625 \times 3125$	$= 1\,953\,125$
$= 1\,953\,125$	

LS = RS

Yes, it would seem that any value of *n* is a correct value.

c. Question 21.a. illustrates the law of multiplication of powers with the same base.

Now Try This

Pattern

22. a. $1^3 = 1 \longrightarrow 1^2$

$1^3 + 2^3 = 9 \longrightarrow 3^2$

$1^3 + 2^3 + 3^3 = 36 \longrightarrow 6^2$

$1^3 + 2^3 + 3^3 + 4^3 = 100 \longrightarrow 10^2$

$1^3 + 2^3 + 3^3 + 4^3 + 5^3 = 225 \longrightarrow 15^2$

base increases by 2

base increases by 3

base increases by 4

base increases by 5

b. According to the pattern in the answer to question 22.a., the sum of the first six cubic numbers would be $21^2 = 441$.

Section 1: Activity 4

1. You evaluate the power first because it is higher in the rule for order of operations than the other operations.

2. a. The calculator automatically performed the operation involving the exponent (3^2) and displayed the result (9).

b. No brackets are required because the calculator will perform all of the operations that follow the subtraction first.

c. The [=] executes the multiplication and subtraction operations and then displays the result.

3. ### With Fraction Key

[1] [$a\frac{b}{c}$] [2] [x^y] [2]

[+] [1] [$a\frac{b}{c}$] [4] [×] [2] [=]

0.75

Without Fraction Key

[1] [÷] [2] [=] [x^y] [2]

[+] [1] [÷] [4] [×] [2] [=]

0.75

4. The method using the fraction key uses one less keystroke.

5. ### Example 4

[1] [0] [x^y] [2] [−] [3] [x^y] [2]

[×] [(] [4] [+] [1] [)] [=]

55.

Example 5

6 ÷ (3 x^y 2 − (2 +/– + 1)) =

0.6

$0.6 = \frac{3}{5}$

6. Most scientific calculators can evaluate a set of operations such as in Example 4 using only one set of brackets. You needed two sets of brackets in Example 5 because two operations of the same level follow the operation involving the exponent.

7. 3 x^y 2 + 5 × 1 +/– x^y 3 = x^y 2 +/– =

0.0625

$0.0625 = \frac{1}{16}$

8. You should find that a scientific calculator can perform this sequence of operations without any brackets.

9. a. $3^3 - 3 \times 2 = 27 - 3 \times 2$
$= 27 - 6$
$= 21$

b. $2^3 - 2^2 = 8 - 4$
$= 4$

c. $\frac{4^3}{4^2 \times 2} = \frac{64}{16 \times 2}$
$= \frac{64}{32}$
$= 2$

d. $\frac{9^2 - (3 \times 6)}{12 - (4+1)} = \frac{9^2 - 18}{12 - 5}$
$= \frac{81 - 18}{12 - 5}$
$= \frac{63}{7}$
$= 9$

e. $\frac{(12-6) \times (12+6)}{(3+3)^2} = \frac{6 \times 18}{6^2}$
$= \frac{\cancel{6}^{1} \times \cancel{18}^{3}}{\cancel{36}_{\cancel{6}_{1}}}$
$= 3$

f. $\frac{3^2 \times 4 + 3 \times 2}{16 \div 4 + 8 + 2} = \frac{9 \times 4 + 3 \times 2}{16 \div 4 + 8 + 2}$
$= \frac{36 + 6}{4 + 8 + 2}$
$= \frac{42}{14}$
$= 3$

g. $2^2+3^2+4^2-(2+3)^2=2^2+3^2+4^2-5^2$

$=4+9+16-25$

$=4$

10. a. 8 +/− x^y 2 − 5 x^y 2 =

39.

b. 3 +/− x^y 2 + 8 +/−

= ÷ 2 +/− = x^y 3 +/− =

-8.

c. 2 +/− × (2 +/− x^y 2

\+ 3 +/− x^y 2

\+ 4 +/− x^y 2) =

-58.

d. 2 × 3 +/− × 1 2 +/− ÷

72.

(1 6 +/− ÷ 2 +/− x^y 4) =

-72.

11. Starting inside the brackets first will reduce the number of keystrokes by 1. The keystrokes can be entered as follows:

2 +/− x^y 2 + 3 +/− x^y 2 +

4 +/− x^y 2 = × 2 +/− =

-58.

12. a. $(3+6^2)\div(14-1)=3$ **b.** $(27+7.8)\div 2-4=13.4$

c. $\left[(4+5)^2+3^2\right]\div 10=9$ **d.** $\dfrac{3\times(4^2+2)}{34-(3+4)}=2$

13. **a.** added 6 and 7 before multiplying

b. multiplied 2 and 3 before applying the exponent

c. just worked from left to right except for the exponent

Now Try This

14. Answers may vary. One possibility is given.

MATH → MATS → MAPS → MOPS → TOPS

Section 1: Activity 5

1. To simplify algebraic expressions, separate the numeric parts and each of the common variable bases. Apply the appropriate exponent rule to reduce the expression to a form with the smallest exponents. Put the variables in alphabetic order.

2. **a.**
$$\begin{aligned}\frac{10x^2y^3}{2xy^2} &= \frac{10}{2}\times\frac{x^2}{x}\times\frac{y^3}{y^2}\\ &= 5x^{2-1}y^{3-2}\\ &= 5x^1y^1 \text{ or } 5xy\end{aligned}$$

b.
$$\begin{aligned}\frac{18x^3y}{6x^3} &= \frac{18}{6}\times\frac{x^3}{x^3}\times y\\ &= 3x^{3-3}y\\ &= 3x^0y\\ &= 3y\end{aligned}$$

c.
$$\begin{aligned}\frac{14a^{-5}b^2}{7a^{-1}b^{-4}} &= \frac{14}{7}a^{-5-(-1)}b^{2-(-4)}\\ &= 2a^{-5+1}b^{2+4}\\ &= 2a^{-4}b^6\\ &= \frac{2b^6}{a^4}\end{aligned}$$

d.
$$\begin{aligned}n^2p^3\left(2n^{-1}p^2\right) &= 2n^{2+(-1)}p^{3+2}\\ &= 2n^1p^5\\ &= 2np^5\end{aligned}$$

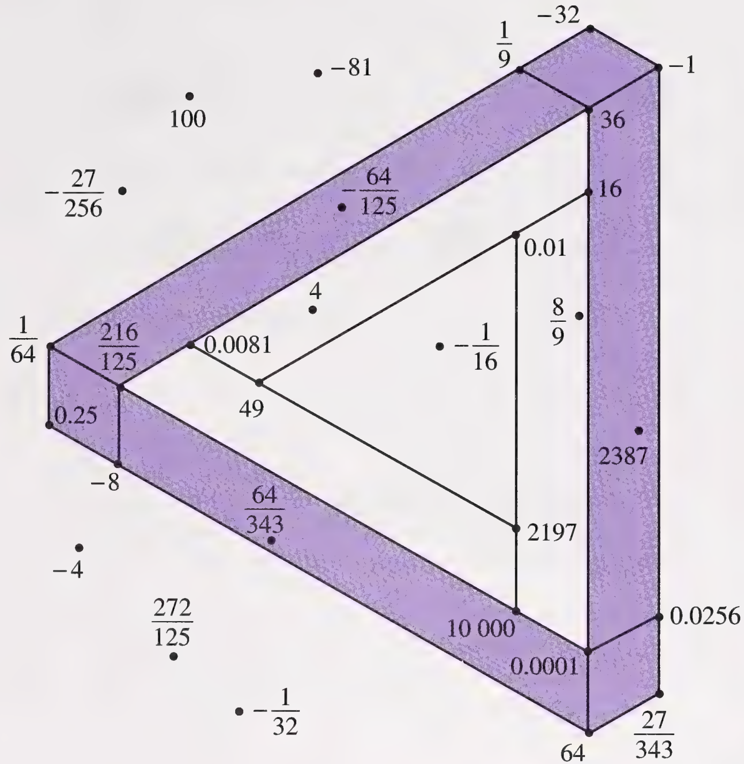
e.
$$\begin{aligned}6a^5b^{-2}\left(3ac^3\right) &= (6\times3)a^{5+1}b^{-2}c^3\\ &= 18a^6b^{-2}c^3 \text{ or } \frac{18a^6c^3}{b^2}\end{aligned}$$

f.
$$\begin{aligned}\frac{12x^2y^3}{7xy} &= \frac{12}{7}x^{2-1}y^{3-1}\\ &= \frac{12}{7}x^1y^2\\ &= \frac{12xy^2}{7}\end{aligned}$$

Section 1: Follow-up Activities

Extra Help

1. The design looks as follows:



2. **a.** $\left(5^3\right)^2 = 5^{3\times2}$ or $\left(5^3\right)^2 = (125)^2$

$$\begin{aligned} \left(5^3\right)^2 &= 5^{3\times2} \\ &= 5^6 \\ &= 15\,625 \end{aligned} \qquad \text{or} \qquad \begin{aligned} \left(5^3\right)^2 &= (125)^2 \\ &= 15\,625 \end{aligned}$$

b.

$$\begin{aligned} \left[(-2)^3\right]^2 &= (-2)^{3\times2} \\ &= (-2)^6 \\ &= 64 \end{aligned} \qquad \text{or} \qquad \begin{aligned} \left[(-2)^3\right]^2 &= (-8)^2 \\ &= 64 \end{aligned}$$

c.

$$\begin{aligned} \left[(1.5)^4\right]^0 &= (1.5)^{4\times0} \\ &= (1.5)^0 \\ &= 1 \end{aligned} \qquad \text{or} \qquad \begin{aligned} \left[(1.5)^4\right]^0 &= (5.0625)^0 \\ &= 1 \end{aligned}$$

d.

$$\begin{aligned} \left(4^3 \times 2^{-3}\right)^2 &= 4^{3\times2} \times 2^{-3\times2} \\ &= 4^6 \times 2^{-6} \\ &= 4096 \times \frac{1}{64} \\ &= 64 \end{aligned}$$

$$\text{or } \begin{aligned} \left(4^3 \times 2^{-3}\right)^2 &= \left(64 \times \frac{1}{8}\right)^2 \\ &= 8^2 \\ &= 64 \end{aligned}$$

e. $\left(\frac{9^2}{3^3}\right)^2 = \frac{9^{2\times2}}{3^{3\times2}}$ or $\left(\frac{9^2}{3^3}\right)^2 = \left(\frac{81}{27}\right)^2$

$= \frac{9^4}{3^6}$ $\quad = 3^2$

$= \frac{6561}{729}$ $\quad = 9$

$= 9$

3. Answers will vary. You may not have noticed any differences in some questions; however, you should have noticed that fewer steps were required in the solutions where the operations in the brackets were done first.

4. a. 11^4 b. $\left(\frac{2}{3}\right)^{-3}$ c. $\left[\left(\frac{1}{8}\right)^{-4}\right]^2$

5. Yes, here is one possible way.

1 $a\frac{b}{c}$ 8 x^y 8 +/– =

6. a. 14641.

b. 3.375

c. 3.^{10}

d. 16777216.

Enrichment

1. Simple interest would be \$480 ($2000 \times 8\% \times 3$) and the amount owed would be \$2480.

2. $A = 750(1.075)^2$

7 5 0 × 1 • 0 7 5 x^y 2 =

866.71875

Alfred owes \$866.72 after two years.

3. $A = 2500(1.0675)^{16}$

2 5 0 0

× 1 • 0 6 7 5 x^y 1 6 =

7109.288652

Mr. Lau will have \$7109.29.

4. **a.** If the interest rate is 7.5%/a and it is calculated semi-annually, then it would only be half that, or 3.75%. Also, there would be eight calculations of interest over the four year period; therefore, the exponent is eight.

b. $A = 800(1.0375)^8$

8 0 0

× 1 • 0 3 7 5 x^y 8 =

1073.976627

Natasha's investment would be \$1073.98 after four years.

c. $A = 800(1.075)^4$

8 0 0

× 1 • 0 7 5 x^y 4 =

1068.375313

Her investment would be \$1068.38. This is \$5.60 less than having her investment compounded semi-annually. The reason is that it is only compounded four times instead of eight; therefore, it is not increasing as quickly.

5. The value 50 appears in cell A5.

6. The amount of Iodine-131 is halved every eight days.

7. The amount of Iodine-131 becomes less than 1 g after about 50 days.

8. Each value increases by a factor of 10.

9. The value in each of cells C4 to C12 is $\frac{1}{1000}$ of the value in the respective cell in column A.

10. **a.** Try $n = 2$.

LS	RS
2^2	2^2

LS = RS

Try $n = 4$.

LS	RS
4^2	2^4
$=16$	$=16$

LS = RS

Therefore, $n = 2$ or $n = 4$.

b.
$$\begin{aligned} 6^6 + 6^6 + 6^6 + 6^6 + 6^6 + 6^6 &= 6^6(1+1+1+1+1+1) \\ &= 6^6 \times 6 \\ &= 6^7 \end{aligned}$$

Therefore, the answer is **B.**

c. Try $n = -2$.

LS	RS
$[-1-(-2)]^3$	1
$=(-1+2)^3$	
$=1^3$	
$=1$	

LS = RS

d. $$4^3 \times 8^2 \times 16^3 = (2^2)^3 \times (2^3)^2 \times (2^4)^3$$
$$= 2^6 \times 2^6 \times 2^{12}$$
$$= 2^{24}$$

e. $$9999^4 \doteq 10\,000^4$$
$$\doteq (10^4)^4$$
$$\doteq 10^{16}$$

Therefore, 9999^4 is between 10^{15} and 10^{16}.

f. $100^{25} = 10^{50}$

$$\begin{array}{r} \overbrace{100\,000\,000\,000\ldots000}^{50\text{ zeros}} \\ -\qquad\qquad\qquad 25 \\ \hline \underbrace{99\,999\,999\,999\ldots9}_{48\text{ nines}}75 \end{array}$$

Determine the sum of the digits.

$$(48 \times 9) + 7 + 5 = 432 + 7 + 5$$
$$= 444$$

The sum of the digits is 444.

g.

$5^1 = 5$

$5^2 = 25$

$5^3 = 125$

$5^4 = 625$

$5^5 = 3\ 125$

$5^6 = 15\ 625$

$5^7 = 78\ 125$

$5^8 = 390\ 625$

The following pattern occurs.

- All odd exponents end in 125.
- All even exponents end in 625.

Therefore, the last three digits are 125.

Section 2: Activity 1

1. a. $100 \div 1000 = \frac{100}{1000}$

   b. $\frac{100}{1000} = \frac{1}{10}$

   c. 1 ÷ 1 0 =

   0.1

   d. $100 = 10^2$ and $1000 = 10^3$

   e. $10^2 \div 10^3 = 10^{2-3}$

   $= 10^{-1}$

   f. Yes, according to the chart, $10^{-1} = \frac{1}{10}$ or 0.1.

2. $10^{-10} = 0.000\,000\,000\,1$

   There are nine zeros between the decimal and the first non-zero digit.

3. The answer to the puzzle is as follows:

   HAUNTED HOUSE: **A NERVOUS MANOR**
   SUIT OF ARMOR: **KNIGHT GOWN**
   WITCH: **FLYING SORCERER**

4. a. $(9\times10^{10})+(7\times10^{9})$

   b. $(8\times10^{-8})+(9\times10^{-9})$

   c. $(3\times10^{6})+(4\times10^{2})+(5\times10^{0})+(6\times10^{-3})$

   d. $(7\times10^{8})+(8\times10^{6})+(9\times10^{-3})$

   e. 1×10^{12}

5. a. six hundred million
   b. six millionths
   c. eight hundred six thousand one and forty-two ten thousandths
   d. one billionth

6. a. 0.054 b. 80 000.03
   c. 1 000 000 000 d. 640 000.008

7.

Expanded Form	Standard Form
8.6×10^{1}	86
8.6×10^{2}	860
8.6×10^{3}	8600
8.6×10^{4}	86 000

Expanded Form	Standard Form
8.6×10^{-1}	0.86
8.6×10^{-2}	0.086
8.6×10^{-3}	0.0086
8.6×10^{-4}	0.000 86

8. The diameter of the virus is 10^{-7} m.

Section 2: Activity 2

1.

Standard Form	Product Form	Scientific Notation
8 640 000	$8.64 \times 1\,000\,000$	8.64×10^{6}
864 000	$8.64 \times 100\,000$	8.64×10^{5}
86 400	$8.64 \times 10\,000$	8.64×10^{4}
8640	8.64×1000	8.64×10^{3}
864	8.64×100	8.64×10^{2}
86.4	8.64×10	8.64×10^{1}
8.64	8.64×1	8.64×10^{0}
0.864	8.64×0.1	8.64×10^{-1}
0.0864	8.64×0.01	8.64×10^{-2}
0.008 64	8.64×0.001	8.64×10^{-3}
0.000 864	8.64×0.0001	8.64×10^{-4}

2. You would not write 8640 in scientific notation because it isn't any more compact in that form.

3. Distance between Saturn and the Sun $= 1\,427\,000\,000$

$$= 1.427 \times 1\,000\,000\,000$$

$$= 1.427 \times 10^{9} \text{ km}$$

Mass of an oxygen atom $= 0.000\,000\,000\,000\,000\,000\,000\,024\,9$

$$= 2.49 \times 0.000\,000\,000\,000\,000\,000\,000\,01$$

$$= 2.49 \times 10^{-23} \text{ g}$$

4. a. 2.4×10^{7} b. 4.3×10^{-7} c. 5.46×10^{10}
d. 3.9×10^{-11} e. 1.47×10^{8} f. 8.3×10^{-8}

5. a. The number is not multiplied by a power of 10.
4.8×10^{2}

b. The number is not between 1 and 10.
5.2×10^{-8}

c. The number is written in scientific notation.

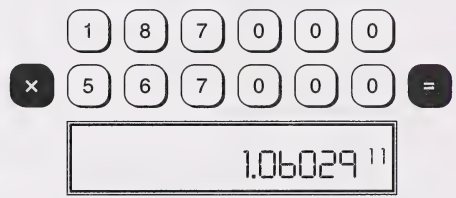
d. The number is written in scientific notation.

e. The number is not between 1 and 10.
4.5×10^{14}

6. a. 4.8×10^{10} b. 7.2×10^{-9} c. 1×10^{15}

7. a. 712 000 000 b. 0.000 000 000 42
c. 100 000

8.



9. [.] [0] [0] [0] [6]

[÷] [3] [0] [0] [0] [0] [=]

$2.^{-08}$

10. **a.** 4.35×10^{6}

b. 3.27×10^{-5}

11. The answer to question 8 is displayed in scientific notation because it has too many digits to display.

12.

9999999999.

Most scientific calculators allow a maximum of ten digits to be entered. Therefore, the largest number is nine billion nine hundred ninety-nine million nine hundred ninety-nine thousand nine hundred ninety-nine.

13. **a.**

0.03

b.

$3.^{-03}$

c.

$3.^{-04}$

d.

0.03

e.

$3.^{-03}$

f.

$3.^{-04}$

14. If the result contains two or more zeros before a non-zero digit, then it is displayed in scientific notation.

15. $13\,000\,000 = 1.3 \times 10\,000\,000$

$= 1.3 \times 10^{7}$

The temperature at the centre of the sun is about $1.3 \times 10^{7}\,°\mathrm{C}$.

16. $0.000\,000\,000\,000\,000\,000\,000\,001\,67$

$= 1.67 \times 0.000\,000\,000\,000\,000\,000\,000\,001$

$= 1.67 \times 10^{-24}$

The mass of a hydrogen atom is about 1.67×10^{-24} g.

17. $4000 \times 30 \times 365 \times 24 = 1\,051\,200\,000$

The number of person-hours of work is 1 051 200 000.

$$\begin{aligned} 1\,051\,200\,000 &= 1.0512 \times 10^9 \\ &= 1.05 \times 10^9 \end{aligned}$$

The number of person-hours of work was approximately 1.05×10^9.

Section 2: Activity 3

1. a. 8 • 1 4 EXP 4 +/– × 9 • 2 5 EXP 6 =

7529.5

In scientific notation, the answer is 7.53×10^3.

b. 7 • 7 3 EXP 5 × 2 • 6 8 EXP 1 +/– =

207164.

In scientific notation, the answer is 2.07×10^5.

c. 3 • 9 1 EXP 1 1 × 1 • 8 4 EXP 1 9 +/– =

7.1944-08

In scientific notation, the answer is 7.19×10^{-8}.

2. Answers will vary depending on the scientific calculator you have.

a. 1 • 2 EXP 6 +/– × 6 • 4 EXP 1 1 =

b. 9 • 7 EXP 2 1 × 3 • 7 EXP 1 4 +/– =

3. a. 7.94×10^{12} b. -3.22×10^6 c. 9.01×10^{-15}

4. $$\begin{aligned} 300\,000\,000 \times 3600 &= \left(3 \times 10^8\right) \times \left(3.6 \times 10^3\right) \\ &= (3 \times 3.6) \times \left(10^8 \times 10^3\right) \\ &= 10.8 \times 10^{11} \\ &= 1.08 \times 10^{12} \end{aligned}$$

1 h = 3600 s

Light travels 1 080 000 000 000 m in one hour.

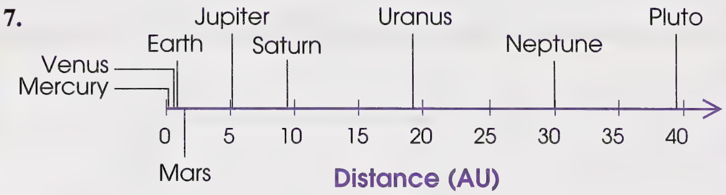
5.

Planet	Mean Distance to Sun (km)	Mean Distance to Sun (AU)
Mercury	5.7900×10^{7}	0.387
Venus	1.0820×10^{8}	0.723
Earth	1.4960×10^{8}	1.000
Mars	2.2790×10^{8}	1.523
Jupiter	7.7840×10^{8}	5.203
Saturn	1.4238×10^{9}	9.517
Uranus	2.8687×10^{9}	19.176
Neptune	4.4921×10^{9}	30.027
Pluto	5.9265×10^{9}	39.616

6. **a.** Saturn is about halfway between the Sun and Uranus.

b. Pluto is approximately 40 times further from the Sun than Earth.

7.



8. Estimate.

$$\frac{1.4 \times 10^{9}}{0.25} = \frac{1.4}{0.25} \times 10^{9}$$
$$= 5.6 \times 10^{9}$$

Enter the keystrokes as follows:

1 • 4 EXP 9 ÷ • 2 5 =

5600000000.

There is about 5.6×10^{9} kg of gold in the oceans.

9. Estimate.

$$\frac{1.427 \times 10^{9}}{1.08 \times 10^{9}} \doteq \frac{1 \times 10^{9}}{1 \times 10^{9}}$$
$$\doteq 1$$

Enter the keystrokes as follows:

1 • 4 2 7 EXP 9
÷ 1 • 0 8 EXP 9 =

1.32 1296296

Therefore, it will take approximately 1.32 h (about 79 min) for light from the Sun to reach Saturn.

10. Estimate.

1 L = 1 kg of water

$$\frac{1}{3\times10^{-26}} \doteq \frac{1}{3}\times10^{26}$$
$$\doteq 0.33\times10^{26}$$
$$\doteq 3.3\times10^{-1}\times10^{26}$$
$$\doteq 3.3\times10^{25}$$

Enter the keystrokes as follows:

1 ÷ 3 EXP 2 6 +/− =

3.333333333 25

One litre of water contains about
33 000 000 000 000 000 000 000 000 molecules of water.

11. a. Estimate.

$$\left(5.98\times10^{11}\right)\div\left(3.03\times10^{7}\right)\doteq\left(6\times10^{11}\right)\div\left(3\times10^{7}\right)$$
$$\doteq(6\div3)\times10^{11-7}$$
$$\doteq 2\times10^{4}$$

Enter the keystrokes as follows:

5 • 9 8 EXP 1 1
÷ 3 • 0 3 EXP 7 =

19735.9736

Each person's share of the national debt is about $19 736.

b. $60\times24\times64\ 459 = 92\ 820\ 960$

The increase is about 9.28×10^{7} (or 93 million) dollars per day.

$60\times24\times30\times64\ 459 = 2\ 784\ 628\ 800$

The increase is about 2.78×10^{9} (or 3 billion) dollars per month.

c. Answers will vary. Your calculation will not be exactly the same as the amount you find on the Internet or in a Statistics Canada book since the rate of increase is always changing.

d. If you find that the rate of increase per minute is decreasing even though the total amount of the debt may still be increasing, then the national debt situation is improving.

12. a. $$\begin{aligned} 20 \div 0.000\,000\,1 &= \left(2\times10^{1}\right)\div\left(1\times10^{-7}\right) \\ &= (2\div1)\times10^{1-(-7)} \\ &= 2\times10^{8} \end{aligned}$$

The computer is 2×10^{8} (or 200 000 000) times faster.

b. There are $365\times24\times60\times60 = 31\,536\,000$ s in one year.

$$\begin{aligned} \therefore\ 2\times10^{8}\div3.1536\times10^{7} &= (2\div3.1536)\times10^{8-7} \\ &= 6.341\,958\,397 \end{aligned}$$

You would need over 6 years.

13. a. 8 • 5 EXP 6

× 7 • 4 EXP 7 ÷

6.29 14

2 • 8 EXP 1 1 =

2246.428571

The answer is 2.25×10^{3}, rounded to two decimal places.

b. 4 EXP 6 × 5 EXP 8

× 3 EXP 7 ÷

6. 22

6 EXP 1 0 =

1. 12

The answer is 1×10^{12}.

c. 4 EXP 7 +/–

× 8 • 5 EXP 7 ÷

34.

1 • 7 EXP 7 +/– =

200000000.

The answer is 2×10^{8}.

d. 4 EXP 1 0

× 3 • 5 EXP 5 +/– ÷

1400000.

2 • 5 EXP 2 +/– =

56000000.

The answer is 5.6×10^{7}.

Now Try This

14. 3 3 × 2 x^y 2 5 9 =

3.0569 1 156 79

In scientific notation, $33 \times 2^{259} \doteq 3.06 \times 10^{79}$.

15. Answers will vary. A sample answer is given based on the human population in November 1996.

a. The world human population is about 6.0×10^{9}.

b. The increase in the human population in 1 min is about 250 people.

c. **After One Year**

$$365 \times 24 \times 60 = 525\,600$$
$$= 5.256 \times 10^{5}$$

There are 5.256×10^{5} min in one year.

$$250 \times 5.256 \times 10^{5} = 2.50 \times 10^{2} \times 5.256 \times 10^{5}$$
$$= 1.314 \times 10^{8}$$

After one year, the population will increase by 1.314×10^{8} people (or 131 400 000 people).

After Ten Years

$$1.314 \times 10^{8} \times 10 = 1.314 \times 10^{9}$$

After ten years, the population will increase by 1.314×10^{9} people (or 1 314 000 000 people).

d. Determine the number of years needed to increase the population by 6.0×10^{9}.

$$\frac{6.0 \times 10^{9}}{1.314 \times 10^{8}} \doteq 45.7$$

It will take about 46 years to double the human population at the present rate of change.

Section 2: Follow-up Activities

Extra Help

1. $(9\times10^{4})\div(3\times10^{-6})$

2. a. $\boxed{2.\ ^{10}}$

   b. $\boxed{3.\ ^{11}}$

   c. $\boxed{4.\ ^{12}}$

3. a. The answer to the puzzle is **A LION THAT MAKES ITS OWN WINE!**

   b. The answer to the puzzle is as follows:

   THE PRICE OF BEEF IS GETTING SO HIGH THESE DAYS THAT MANY HOT DOG MAKERS ARE FINDING IT VERY HARD TO MAKE BOTH ENDS MEAT.

   c. The answer to the puzzle is **THERES NO POLICE LIKE HOLMES.**

   d. The answer to the puzzle is **THE TWO GUYS WHO ROBBED A MUSIC STORE AND GOT AWAY WITH THE LUTE.**

Enrichment

1. a. billion b. trillion c. quadrillion
   d. quintillion e. sextillion f. septillion
   g. octillion h. nonillion i. decillion

2. a. 10^{36} b. 10^{39} c. 10^{42}
   d. 10^{45} e. 10^{48} f. 10^{51}
   g. 10^{54} h. 10^{57} i. 10^{60}

3. a. 10^{16} b. 10^{35} c. 10^{10}

4. $$\begin{aligned}43\text{ quintillion} &= 43\times10^{18}\\ &= 4.3\times10^{19}\end{aligned}$$

Can You Build This?

Figure out the value of each expression below and find it next to a dot. Connect the dots in the same order as the exercises are numbered. Be careful to lift your pencil and begin again each time you see the instruction "Lift Pencil". You will create an interesting design.

1. 7^2
2. 2^4
3. $(-6)^2$
4. $\left(\frac{1}{3}\right)^2$

Lift Pencil

1. $(-2)^6$
2. $(-8)^1$
3. $\left(\frac{6}{5}\right)^3$
4. $(0.3)^4$

Lift Pencil

1. $(-1)^7$
2. $\left(\frac{2}{5}\right)^4$
3. $(-0.1)^4$
4. 16^1

Lift Pencil

1. $(-2)^3$
2. $(0.5)^2$
3. $\left(\frac{1}{4}\right)^3$
4. $(-10)^4$
5. $(0.1)^2$

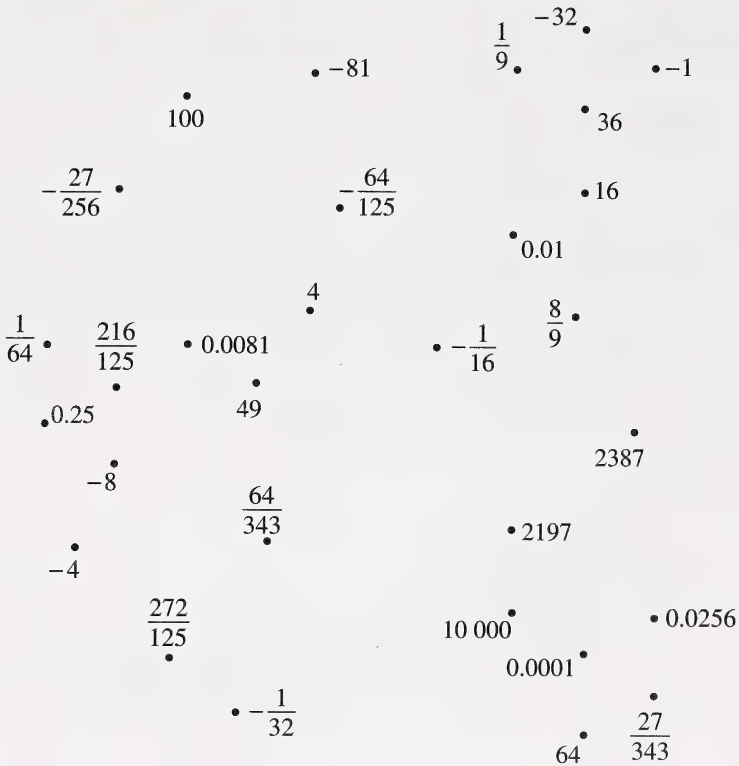
Lift Pencil

1. $(-0.16)^2$
2. $\left(\frac{3}{7}\right)^3$
3. 4^3
4. $(0.01)^2$
5. $(-100)^2$

Lift Pencil

1. 13^3
2. $(-0.09)^2$
3. $(-1)^{19}$
4. $(-2)^5$
5. $\left(\frac{1}{8}\right)^2$

Stop



Daffynition Decoder

HAUNTED HOUSE:

____

10^{-11} 10^{-4} 10^{-7} 10^{4} 10^{-8} 10^{-5} 10^{5} 10^{-2} 10^{3} 10^{-11} 10^{-4} 10^{-5} 10^{4}

SUIT OF ARMOR:

____

10^{9} 10^{-4} 10^{-1} 10^{-13} 10^{2} 10^{6} 10^{-13} 10^{-5} 10^{-6} 10^{-4}

WITCH:

____

10^{12} 10^{7} 10^{0} 10^{-1} 10^{-4} 10^{-13} 10^{-2} 10^{-5} 10^{4} 10^{-3} 10^{-7} 10^{4} 10^{-7} 10^{4}

To decode these three daffynitions, write any expression below as a single power of 10. Each time your answer appears in the code, write the letter of that exercise above it. Keep working and you will decode three de-fun-itions.

T $10^{-2} \times 10^{8} =$

A $10^{-4} \times 10^{-7} =$

U $10^{-3} \times 10^{8} =$

I $10 \times \frac{1}{100} =$

C $\frac{1}{10} \times \frac{1}{100} =$

W $\frac{1}{1\,000\,000} =$

H $10^{9} \div 10^{7} =$

O $10 \div 10^{6} =$

Y $\frac{10^{-2}}{10^{-2}} =$

E $\frac{1}{10\,000} \div 1000 =$

M $\frac{1}{100} \div \frac{1}{100\,000} =$

K $1\,000\,000\,000 =$

V $\frac{1}{100\,000} \times \frac{1}{1000} =$

F $10^{8} \div 10^{-4} =$

S $10^{6} \times 10^{-8} =$

L $\frac{10^{3}}{10^{-4}} =$

G $10^{-10} \times 10^{-3} =$

R $1000 \div \frac{1}{10} =$

N $\frac{10^{2}}{10^{6}} =$

What Has Long Hair and Purple Feet?

Express the number in each statement below in scientific notation. Select your answer from the two choices given. Write the letter of the correct choice in each box at the bottom of the page that contains the statement number.

1. The average distance from the surface of the earth to the surface of the moon is about 376 000 km.

   (E) 3.76×10^{5} (R) 3.76×10^{4}

2. The tallest structure in the world is the Warszawa Radio Mast in Plock, Poland, completed in May 1974. It measures about 650 m height.

   (S) 6.5×10^{2} (A) 65×10^{2}

3. The heaviest bell in the world is the Tsar Kolokoi, cast in 1733 in Moscow. It weighs about 196 000 kg.

   (I) 1.96×10^{3} (H) 1.96×10^{5}

4. One light-year, the distance travelled by light in one year, is about 9 460 000 000 000 km.

   (T) 9.46×10^{10} (O) 9.46×10^{12}

5. Sirius A (the Dog Star) is the brightest star visible in the heavens. It has a mass of about 46 500 000 000 000 000 000 000 000 000 000 kg.

   (D) 4.65×10^{30} (K) 4.65×10^{31}

6. The nearest star beyond our sun is the very faint Proxima Centauri, which is about 40 300 000 000 000 km from the earth.

   (T) 4.03×10^{13} (S) 4.03×10^{11}

7. The smallest known insects are the "hairy-winged" beetles of the Trichopterygidae family. They measure less than 0.02 cm in length.

   (B) 2×10^{-3} (M) 2×10^{-2}

8. The wavelength of yellow light is about 0.000 058 cm.

   (A) 5.8×10^{-5} (U) 5.8×10^{-4}

9. The wavelength of one type of X-ray is about 0.000 000 012 8 cm.

   (N) 1.28×10^{-8} (L) 1.28×10^{-10}

10. The smallest identified virus is the potato spindle tuber virus, which has a diameter of less than 0.000 002 cm.

    (I) 2×10^{-6} (U) 2×10^{-5}

11. One of the least stable atomic particles is the rho prime meson, which has a lifetime of about 0.000 000 000 000 000 000 000 001 6 s.

    (L) 1.6×10^{-24} (P) 1.6×10^{-23}

12. The mass of an electron is about 0.000 000 000 000 000 000 000 000 000 9 g.

    (F) 9×10^{-26} (W) 9×10^{-28}

8	11	10	4	9	6	3	8	6	7	8	5	1	2	10	6	2	4	12	9	12	10	9	1	!

Find a Match

Each of the two blocks below is divided into 25 boxes. Boxes in the top block contain numbers in scientific notation. Express any of these numbers as a decimal number and find this decimal number in the bottom block. Write the word from the top box in the bottom box. Work doggedly and you will spell out a hot message!

8×10^{2} THAT	8×10^{-4} OF	8×10^{-9} BOTH	8×10^{6} ARE	8×10^{-12} THE
8×10^{-11} MANY	8×10^{10} TO	8×10^{9} THESE	4.5×10^{-2} VERY	4.5×10^{4} IS
4.5×10^{-8} GETTING	4.5×10^{11} IT	4.5×10^{-3} SO	4.5×10^{-6} DAYS	4.5×10^{7} DOG
4.5×10^{5} HARD	1.23×10^{3} HOT	1.23×10^{-1} BEEF	1.23×10^{7} MAKERS	1.23×10^{-10} FINDING
1.23×10^{8} ENDS	1.23×10^{-5} PRICE	1.23×10^{9} HIGH	1.23×10^{-7} MEAT	1.23×10^{-6} MAKE
0.000 000 000 008	0.000 012 3	0.0008	0.123	45 000
0.000 000 045	0.0045	1 230 000 000	8 000 000 000	0.000 004 5
800	0.000 000 000 08	1230	45 000 000	12 300 000
8 000 000	0.000 000 000 123	450 000 000 000	0.045	450 000
80 000 000 000	0.000 001 23	0.000 000 008	123 000 000	0.000 000 123

What Did Dr. Watson Say About Sherlock?

Work any problem below and find your answer in the answer columns. Write the letter of the answer in each box at the bottom of the page that contains the problem number. Keep working and you will discover the answer to the title question.

1. $(4\times10^{5})(2\times10^{3})$
2. $(3\times10^{-4})(3\times10^{7})$
3. $(9\times10^{3})(7\times10^{-1})$
4. $(6\times10^{-5})(7\times10^{-2})$
5. $(8\times10^{-13})(5\times10^{4})$
6. $(3.6\times10^{6})(2\times10^{3})$
7. $(5\times10^{-12})(8.1\times10^{15})$
8. $(7.6\times10^{-4})(6\times10^{-4})$
9. $(3.5\times10^{1})(4.5\times10^{-10})$
10. $(6.8\times10^{-18})(2.5\times10^{-12})$

11. Light travels at a speed of 3×10^{5} km/s. Light from Sirius A, the brightest star in the heavens, takes 2.7×10^{8} s to reach the earth. What is the distance to Sirius A?

12. In his book, *Six-Legged Science*, Brian Hocking estimates that the insect population of the world is at least 1×10^{18}. If the average weight of each insect is 2.5×10^{-3} g, what is the total weight of the insect population?

13. The human population of the world is estimated at 4.5×10^{9}. If the average weight of each human is 5.5×10^{4} g, what is the total weight of the human population?

(L) 1.7×10^{-30}		(E) 1.575×10^{-10}	
(R) 4.05×10^{4}		(K) 9×10^{3}	
(V) 8×10^{7}		(D) 7.2×10^{8}	
(A) 8.1×10^{14} km		(H) 1.7×10^{-29}	
(B) 4.2×10^{-5}		(S) 6.3×10^{3}	
(N) 4.2×10^{-6}		(P) 2.5×10^{15} g	
(I) 8×10^{8}		(B) 2.5×10^{5}	
(Y) 4.05×10^{5}		(M) 7.2×10^{9}	
(T) 4.56×10^{-7}		(A) 2.475×10^{13} g	
(E) 8.1×10^{13} km		(O) 1.575×10^{-8}	
(A) 4×10^{-9}		(L) 2.475×10^{14} g	
(C) 4×10^{-8}		(G) 4.56×10^{-8}	

8	10	11	7	11	3	4	9	12	9	13	1	5	11	13	1	2	11	10	9	13	6	11	3

Did You Hear About...

A	B	C	D	E	F	G
H	I	J	K	L	M	N

Directions: Solve each problem in order. Find your answer in one of the answer columns and notice the word next to it. Write this word in the box that contains the same letter as the problem. Keep working and you will hear about something noteworthy!

(A) $\dfrac{9\times10^{6}}{3\times10^{2}}$

(B) $\dfrac{8\times10^{3}}{2\times10^{9}}$

(C) $\dfrac{6\times10^{-1}}{3\times10^{4}}$

(D) $\dfrac{4.8\times10^{-7}}{4\times10^{-5}}$

(E) $\dfrac{7.5\times10^{8}}{5\times10^{-2}}$

(F) $\dfrac{3.5\times10^{-3}}{7\times10^{-9}}$

(G) $\dfrac{6.4\times10^{3}}{8\times10^{4}}$

(H) $\dfrac{4.5\times10^{-6}}{1.5\times10^{2}}$

(I) $\dfrac{7.2\times10^{-10}}{1.8\times10^{-3}}$

(J) $\dfrac{4\times10^{5}}{8\times10^{2}}$

(K) $\dfrac{3\times10^{3}}{1.5\times10^{-7}}$

(L) $\dfrac{8\times10^{-1}}{1.6\times10^{-8}}$

(M) Jupiter, the largest planet in our solar system, is 7.8×10^{8} km from the sun. The speed of light is 3×10^{5} km/s. How many seconds does it take sunlight to reach Jupiter?

(N) The total length of all the drawers in a library card catalogue is 5×10^{3} cm. If each card has a thickness of 2.5×10^{-2} cm, how many cards will fit in the card catalogue?

2×10^{10} – AWAY
2×10^{12} – FIVE
2.6×10^{4} – TEN
8×10^{-2} – MUSIC
1.5×10^{12} – WERE
1.2×10^{-2} – WHO
2×10^{-4} – TIGERS
3×10^{5} – A
4×10^{-6} – TWO
5×10^{-1} – CASH
5×10^{2} – GOT
2.6×10^{3} – THE
3×10^{-8} – STORE
4×10^{-5} – FOR

1.2×10^{-1} – FROM
1.5×10^{10} – ROBBED
5×10^{-6} – TEN
3×10^{4} – THE
3×10^{-10} – HORN
4×10^{-7} – AND
5×10^{7} – WITH
4×10^{-5} – BIG
2×10^{4} – BED
2×10^{5} – LUTE
6×10^{2} – DOUGH
2×10^{-5} – GUYS
5×10^{5} – A
8×10^{2} – BIG

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